

Design and Cryptanalysis of Symmetric-Key Algorithms in Black and White-box Models

Aleksei Udovenko

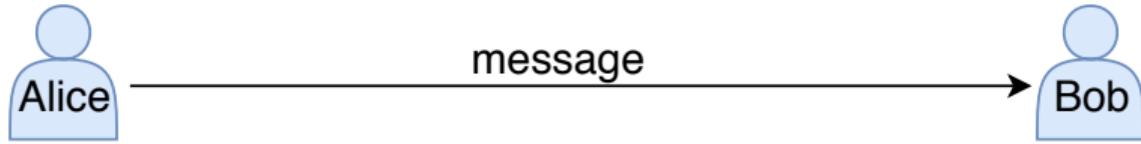
SnT, University of Luxembourg

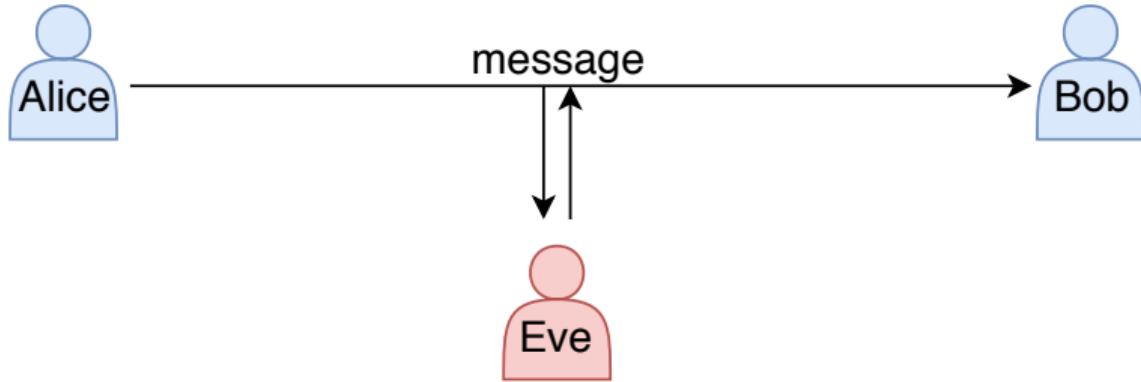
April 9, 2019
PhD Defense

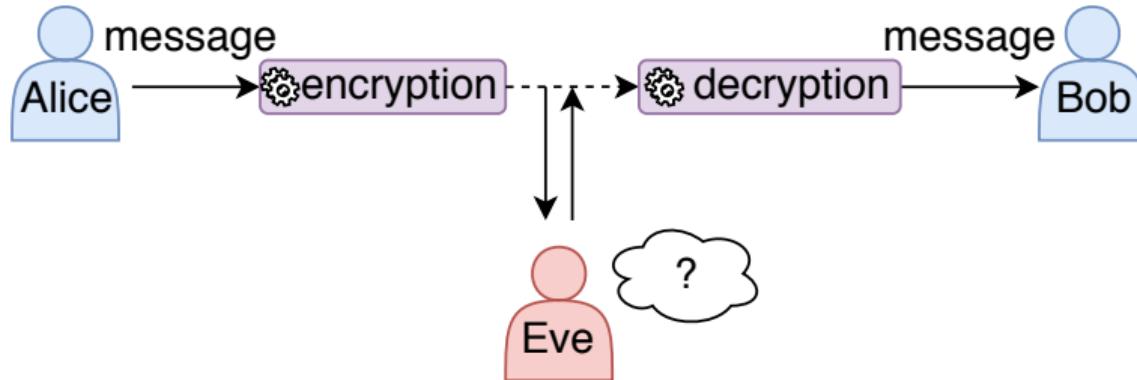


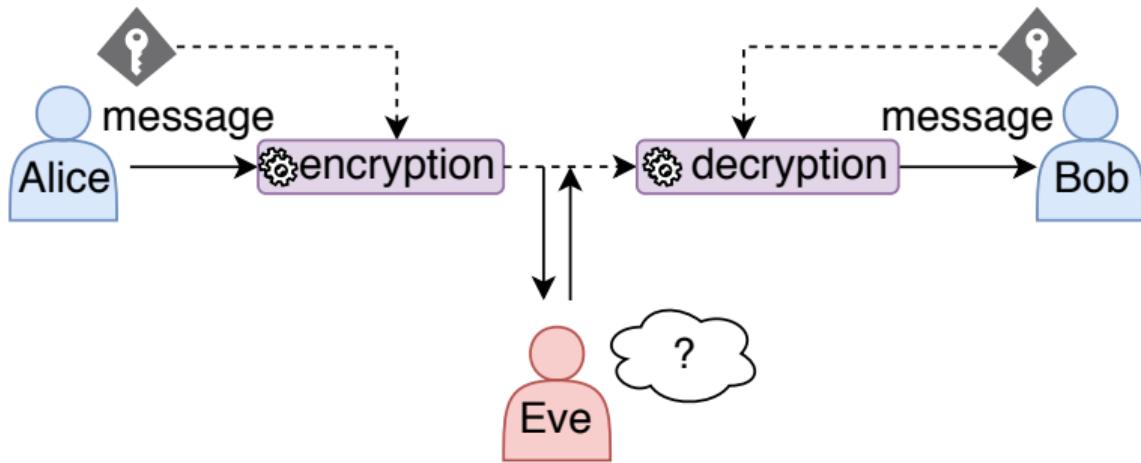
Luxembourg National
Research Fund

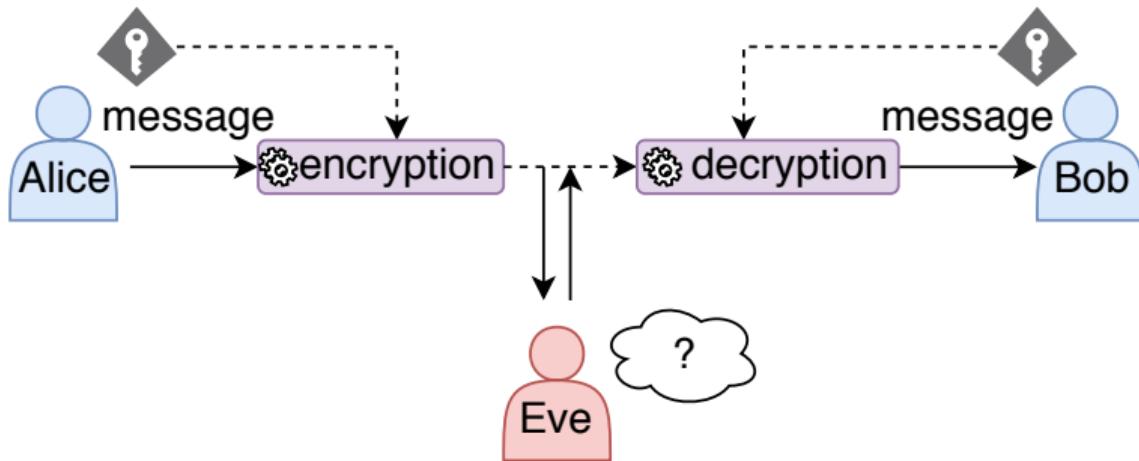




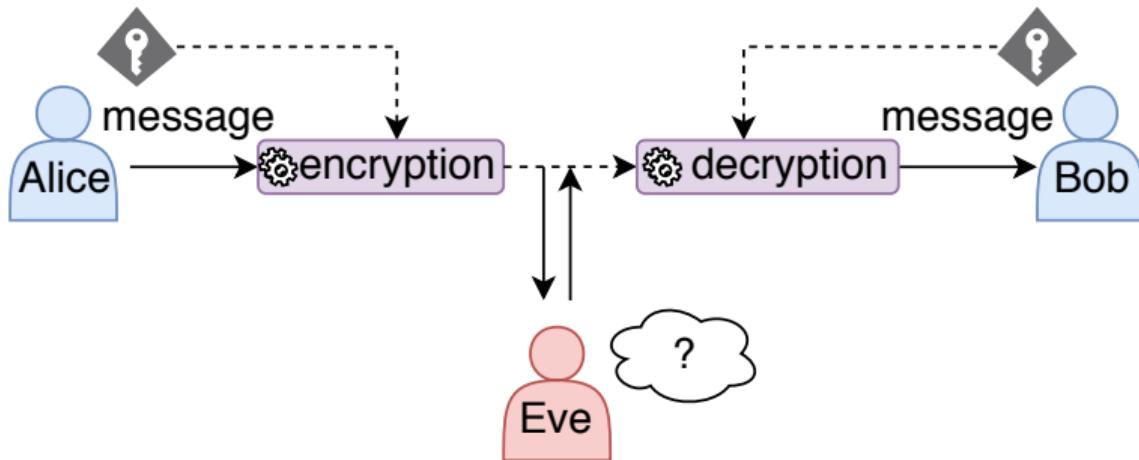








Symmetric-key Cryptography



Symmetric-key Cryptography

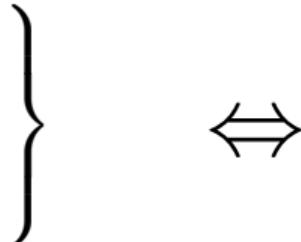
ensures that the message is:

- ① secret (**confidentiality**)
- ② unmodified (**integrity**)
- ③ from the correct person (**authenticity**)

(confidentiality)
(integrity)
(authenticity)



(confidentiality)
(integrity)
(authenticity)



Authenticated
Encryption

(confidentiality)
(integrity)
(authenticity)

}

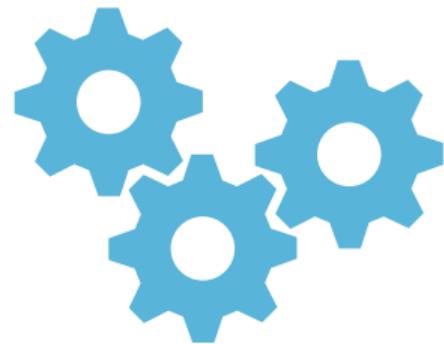


Authenticated
Encryption

The main goal of
symmetric-key cryptography!



How does it work?

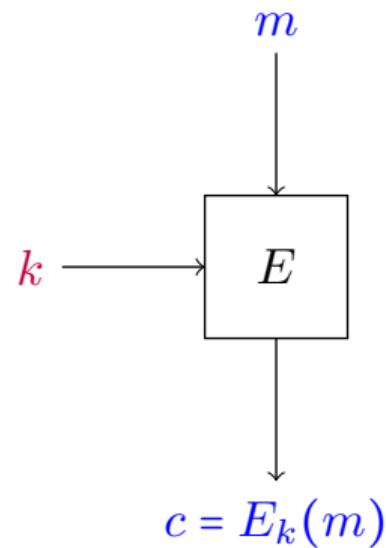


Construction 1:
Block Cipher + Mode of Operation

Block Cipher

An Algorithm E_k :

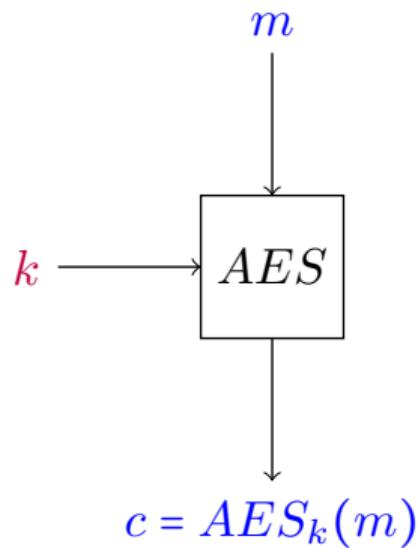
- n -bit message m
- κ -bit key k
- n -bit ciphertext c
- E_k is invertible



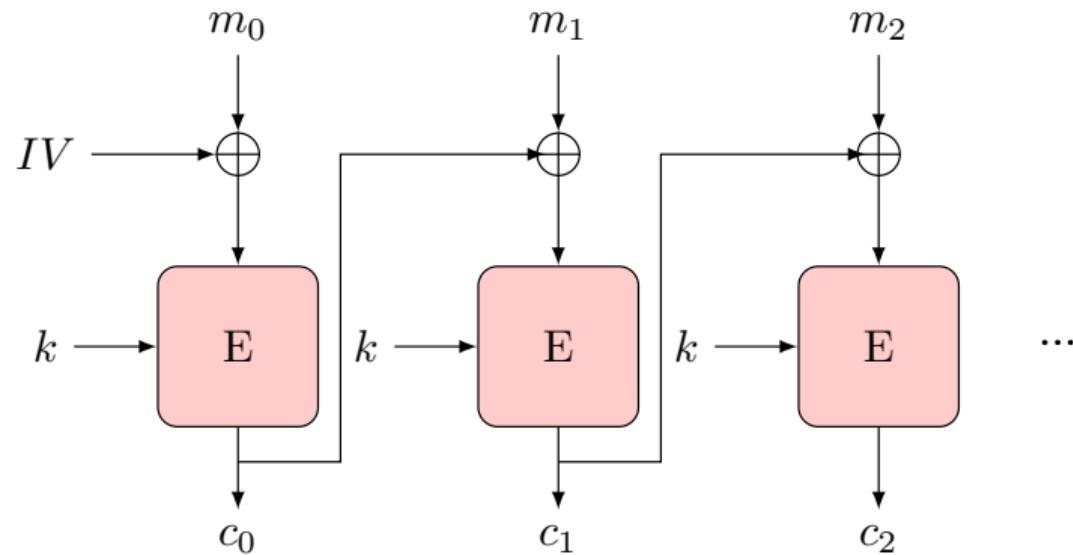
Example: Advanced Encryption Standard

AES Algorithm:

- 128-bit message m
- 128/192/256-bit key k
- 128-bit ciphertext c
- designed in 1998
by V. Rijmen and J. Daemen



Mode of Operation



Example: COLM Mode of Operation

One of CAESAR competition winners (2019)

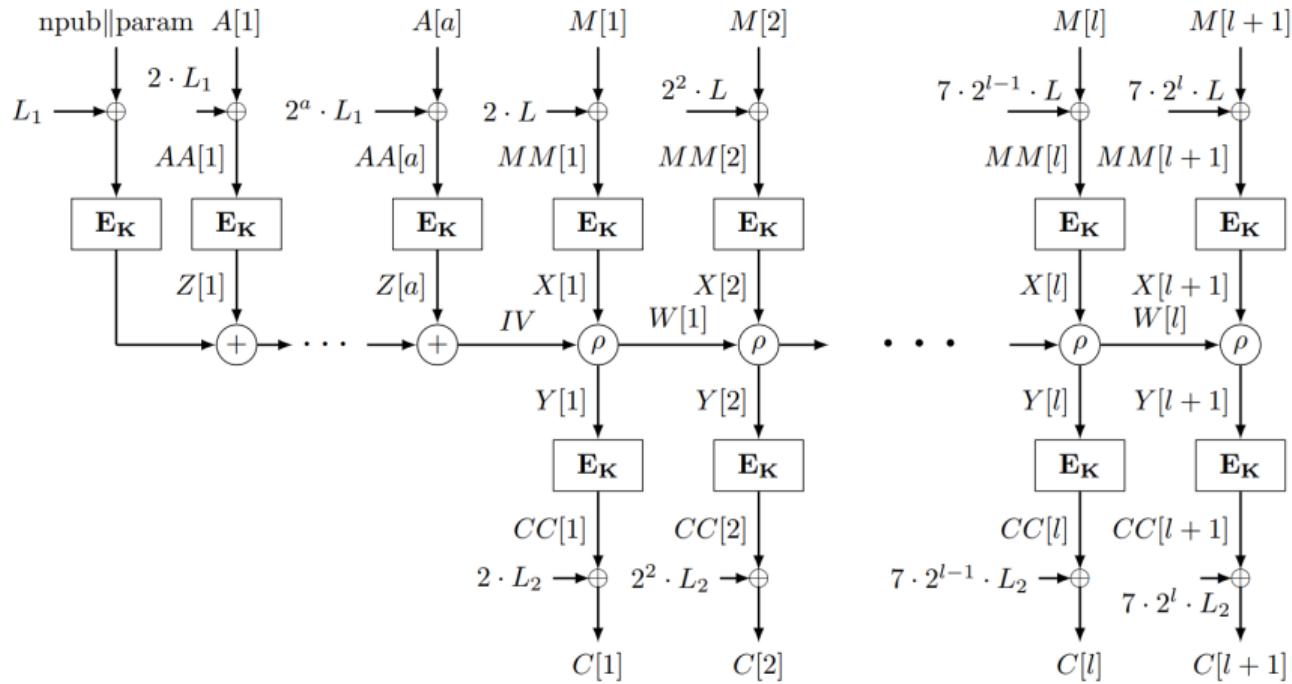
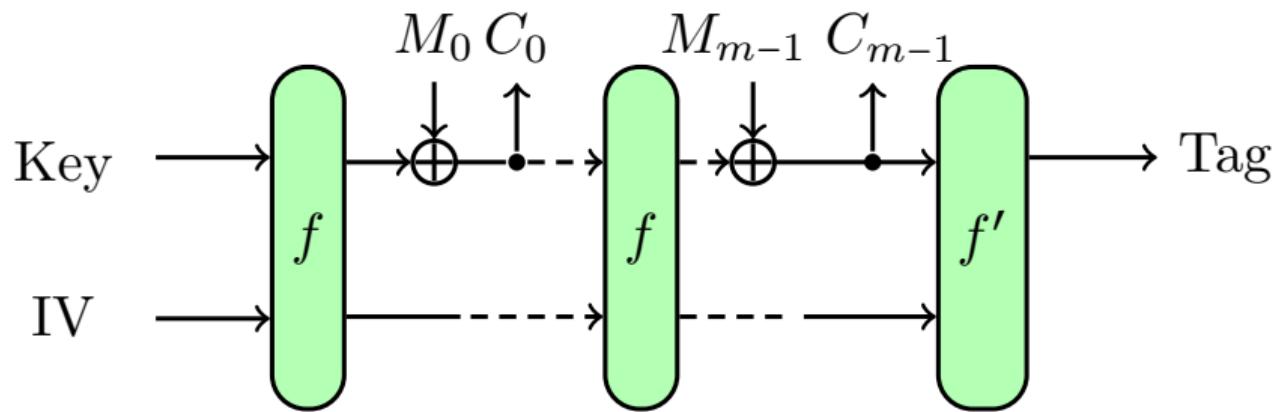


Figure credits: COLM v1 Specification

Construction 2: Sponge Structure

(Duplexed) Sponge Structure



f : keyless invertible function (permutation)

Plan

1 Introduction

2 Thesis Overview

- Design of Symmetric-key Algorithms
- Structural and Decomposition Cryptanalysis
- Nonlinear Invariant Cryptanalysis
- White-box Cryptography

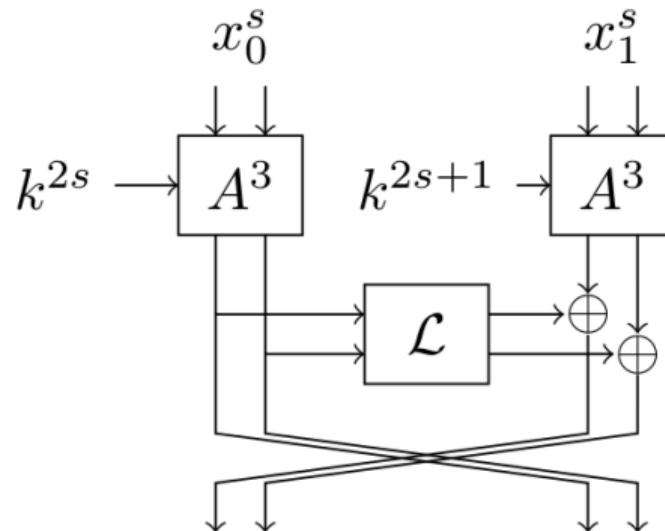
3 White-box Cryptography

Design of Symmetric-key Algorithms

Lightweight Cryptography:

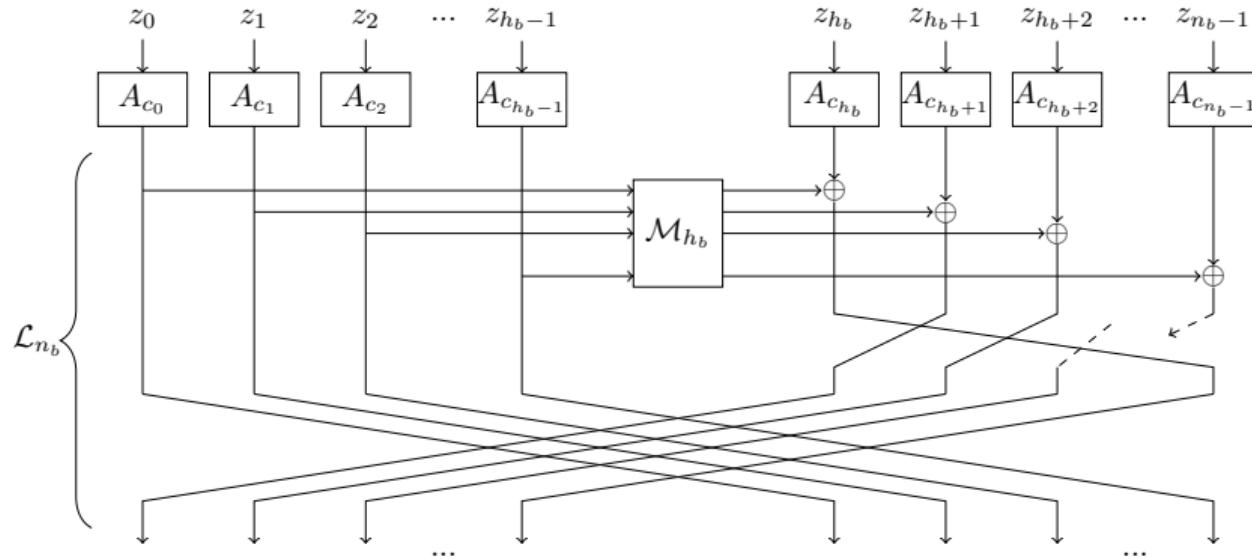
Cryptography for **resource-constrained** devices
(Internet of Things)

Design of Symmetric-key Algorithms



Sparx: a *lightweight* block cipher
based on a **new design strategy**

Design of Symmetric-key Algorithms



Sparkle, Esch and Schwaemm:
cryptographic permutations, hash functions
and authenticated encryption

Design of Symmetric-key Algorithms

-  Daniel Dinu, Léo Perrin, Aleksei Udovenko, Vesselin Velichkov, Johann Großschädl, and Alex Biryukov.
Design Strategies for ARX with Provable Bounds: Sparx and LAX.
In *Advances in Cryptology - ASIACRYPT 2016*, pages 484–513.
<https://www.cryptolux.org/index.php/SPARX>.
-  Christof Beierle, Alex Biryukov, Luan Cardoso dos Santos, Johann Großschädl, Léo Perrin, Aleksei Udovenko, Vesselin Velichkov, and Qingju Wang.
Schwaemm and Esch: Lightweight Authenticated Encryption and Hashing using the Sparkle Permutation Family, 2019.
<https://www.cryptolux.org/index.php/Sparkle>.

How to make sure
that an encryption scheme is **secure**?

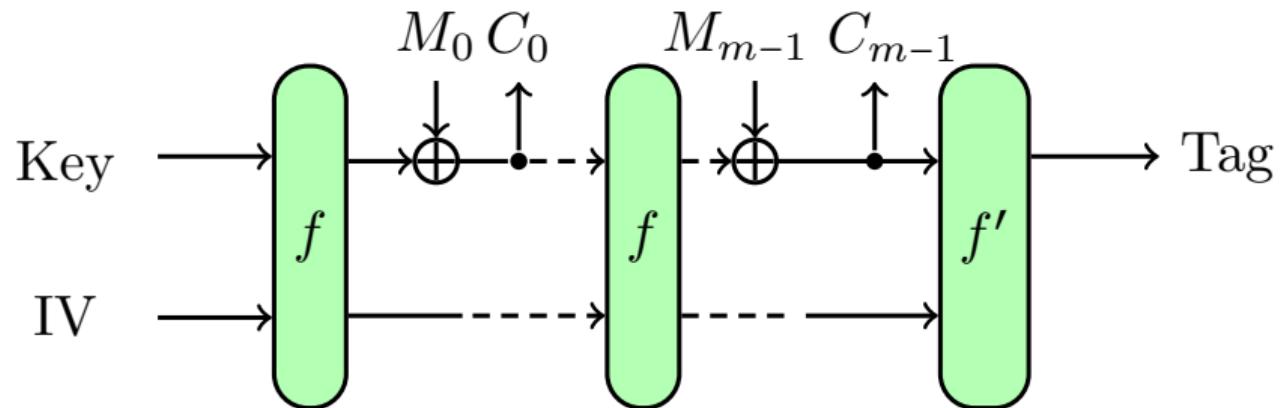


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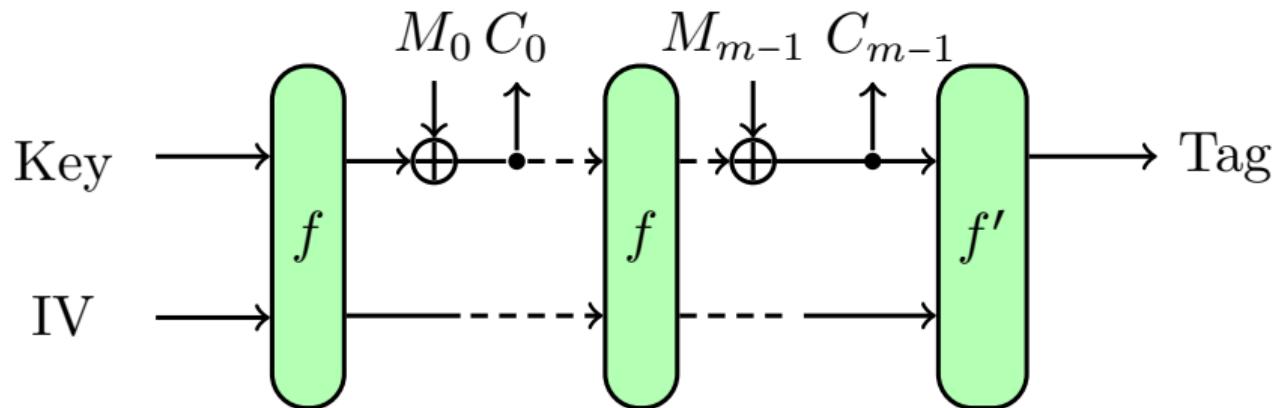


Security Proofs and **Cryptanalysis**!

Security Proofs: Modes and Structures



Security Proofs: Modes and Structures



secure if the permutation f is secure (random)

Cryptanalysis:

an attempt to invalidate
security claims of a cryptosystem
by developing an **attack**

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an attempt to invalidate
security claims of a cryptosystem
by developing an **attack**

- a large variety of methods: differential, linear, integral, ...
- attacks on simplified versions
- analysis of components

Structural and Decomposition Cryptanalysis

Distinguishing structures and recovering components

Structural and Decomposition Cryptanalysis

x

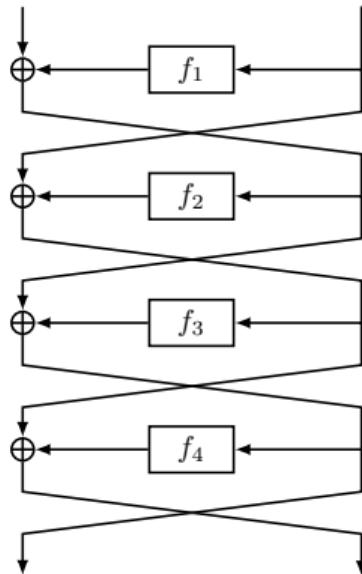


$E(x)$

Structural and Decomposition Cryptanalysis

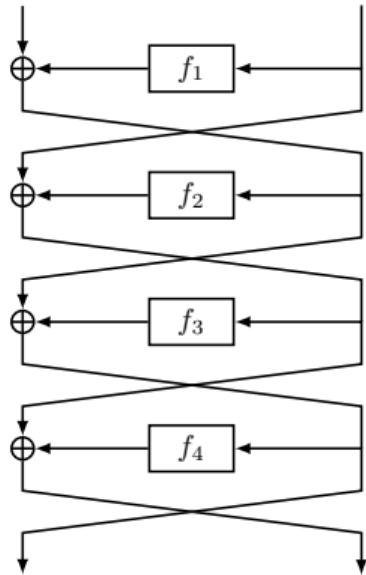
x	$E(x)$
0	182
1	210
2	78
3	251
4	97
...	
252	112
253	19
254	224
255	74

Structural and Decomposition Cryptanalysis



Feistel Networks

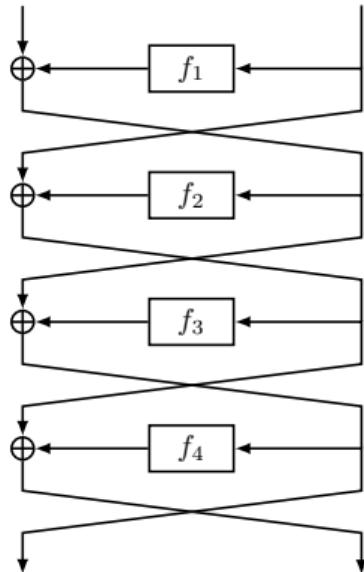
Structural and Decomposition Cryptanalysis



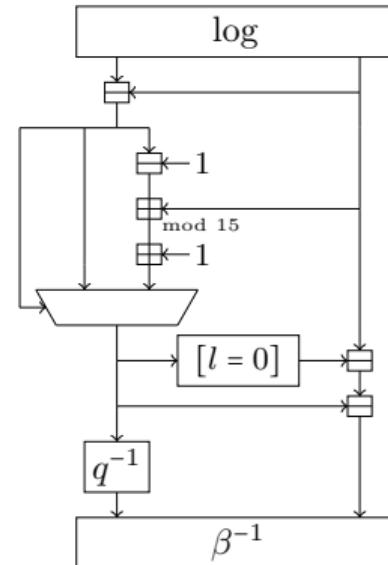
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Feistel Networks

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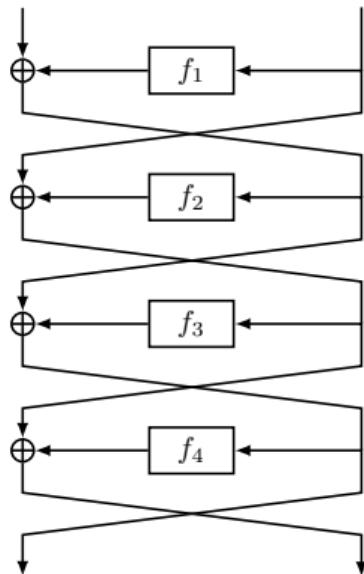


Feistel Networks



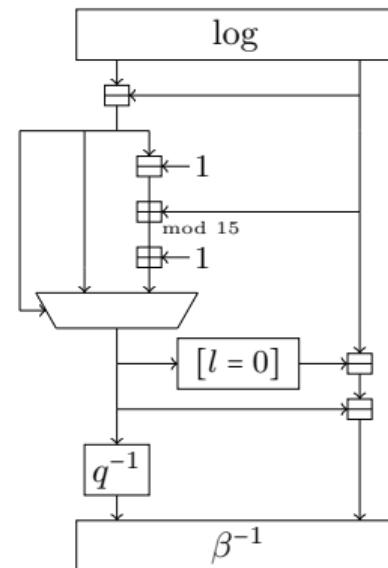
GOST S-Box

Structural and Decomposition Cryptanalysis



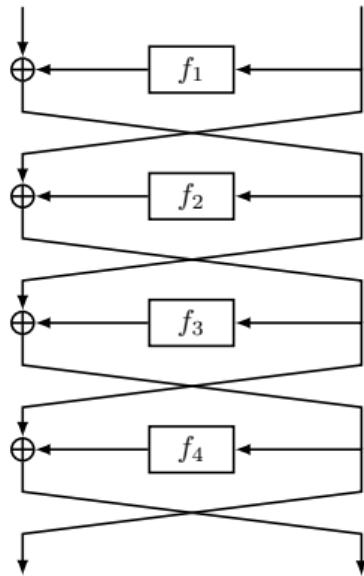
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Feistel Networks

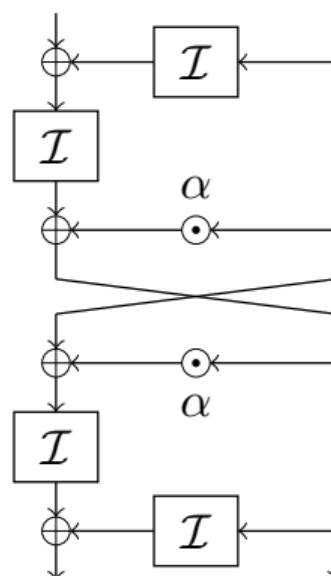


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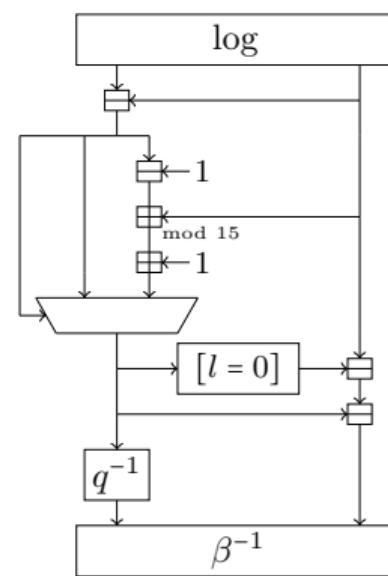
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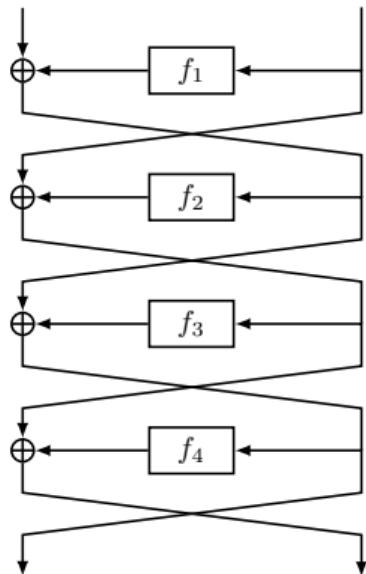


6-bit APN
Permutation

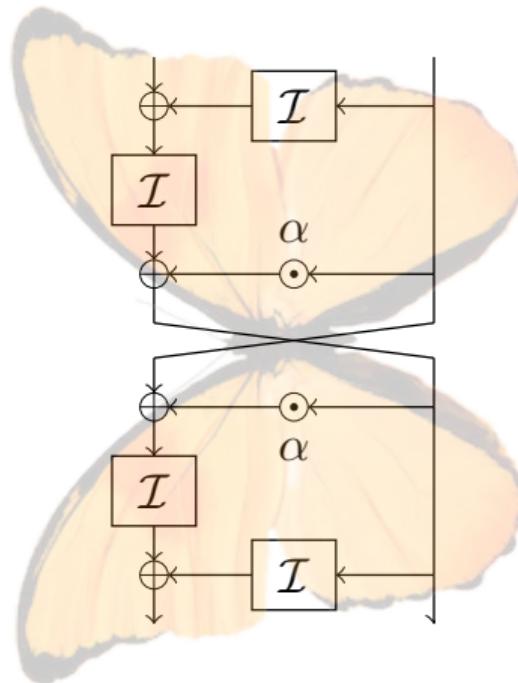


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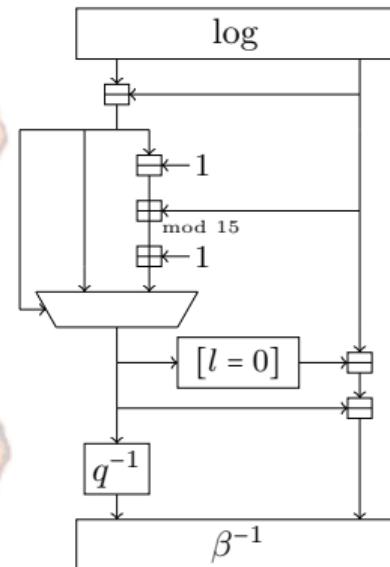
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Feistel Networks



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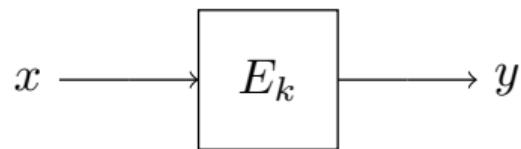
-  Léo Perrin and Aleksei Udovenko.
Algebraic Insights into the Secret Feistel Network.
In *Fast Software Encryption - FSE 2016*, pages 378–398.
-  Léo Perrin, Aleksei Udovenko, and Alex Biryukov.
Cryptanalysis of a Theorem: Decomposing the Only Known Solution to the Big APN Problem.
In *Advances in Cryptology - CRYPTO 2016*, pages 93–122.
-  Alex Biryukov, Léo Perrin, and Aleksei Udovenko.
Reverse-Engineering the S-Box of Streebog, Kuznyechik and STRIBOBr1.
In *Advances in Cryptology - EUROCRYPT 2016*, pages 372–402.
-  Léo Perrin and Aleksei Udovenko.
Exponential S-Boxes: a Link Between the S-Boxes of BelT and Kuznyechik/Streebog.
IACR Trans. Symmetric Cryptol., 2016(2):99–124.

Nonlinear Invariant Cryptanalysis

Properties of messages that are preserved through encryption

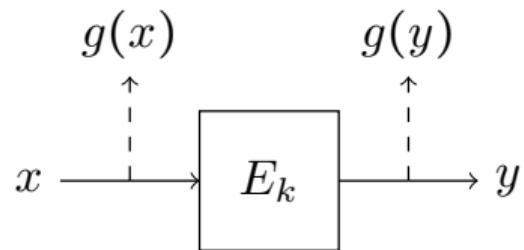
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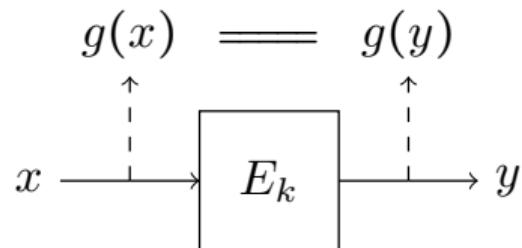
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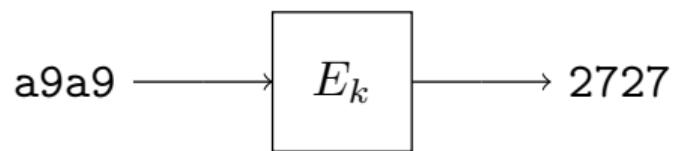
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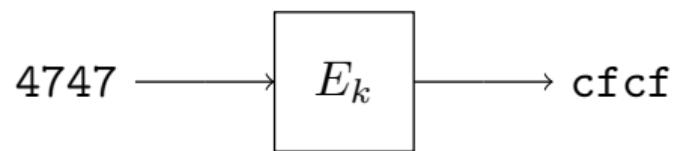
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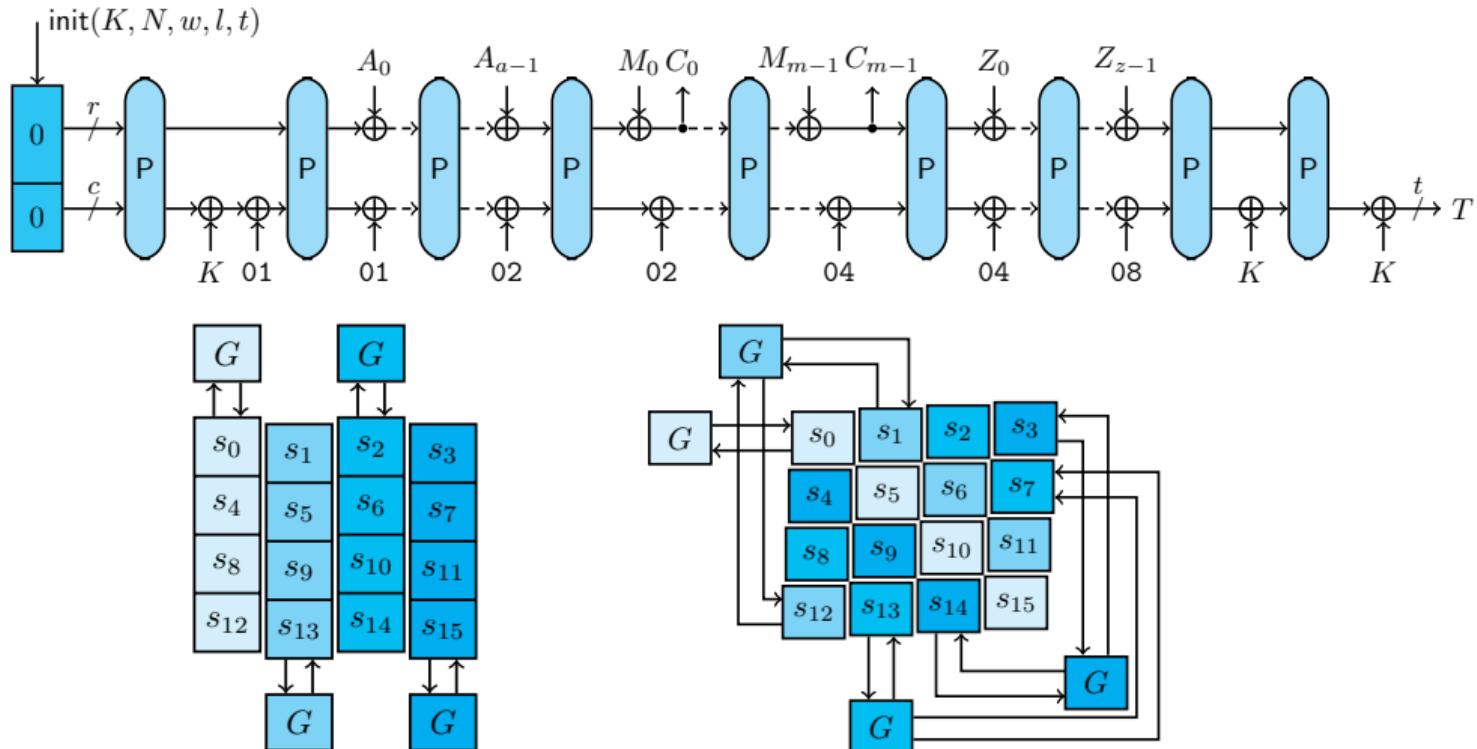


Nonlinear Invariant Cryptanalysis

Properties of messages that are preserved through encryption



Nonlinear Invariant Cryptanalysis



Analysis of the NORX Authenticated Encryption

Figure credits: TikZ for Cryptographers

Nonlinear Invariant Cryptanalysis

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Theoretical study of linear layers
preserving degree- d invariants

Nonlinear Invariant Cryptanalysis

-  [Alex Biryukov, Aleksei Udovenko, and Vesselin Velichkov.](#)
[Analysis of the NORX Core Permutation.](#)
[Cryptology ePrint Archive, Report 2017/034, 2017.](#)
[https://eprint.iacr.org/2017/034.](https://eprint.iacr.org/2017/034)
-  [Christof Beierle, Alex Biryukov, and Aleksei Udovenko.](#)
[On Degree-d Zero-Sum Sets of Full Rank.](#)
[Cryptology ePrint Archive, Report 2018/1194, 2018.](#)
[https://eprint.iacr.org/2018/1194.](https://eprint.iacr.org/2018/1194)

Plan

- 1 Introduction
- 2 Thesis Overview
- 3 White-box Cryptography
 - Introduction
 - Attack Methods
 - Countermeasures

White-box Cryptography

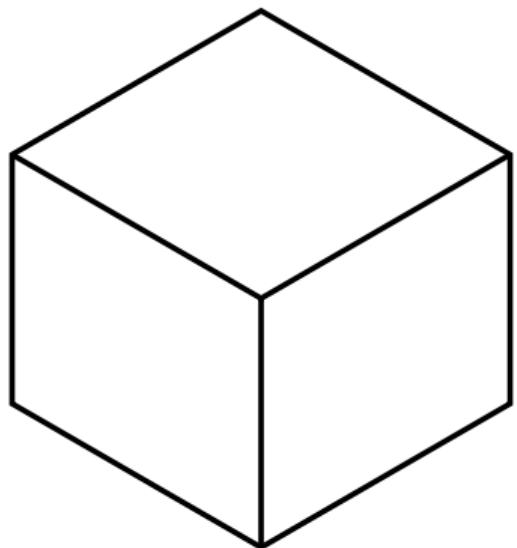


Alex Biryukov and Aleksei Udovenko.

Attacks and Countermeasures for White-box Designs.

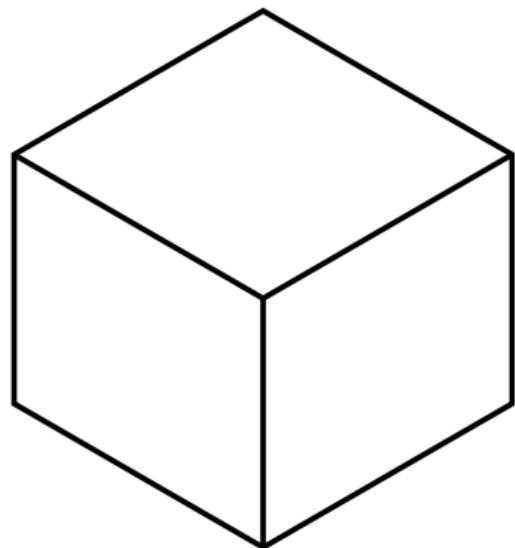
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White-box model



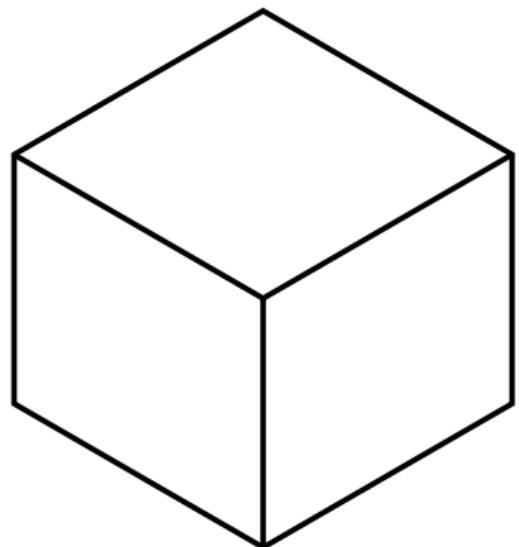
White-box model

- implementation is fully **available** to an adversary
- secret key should be **unextractable**
- **extra**: one-wayness, incompressibility, traitor traceability, ...

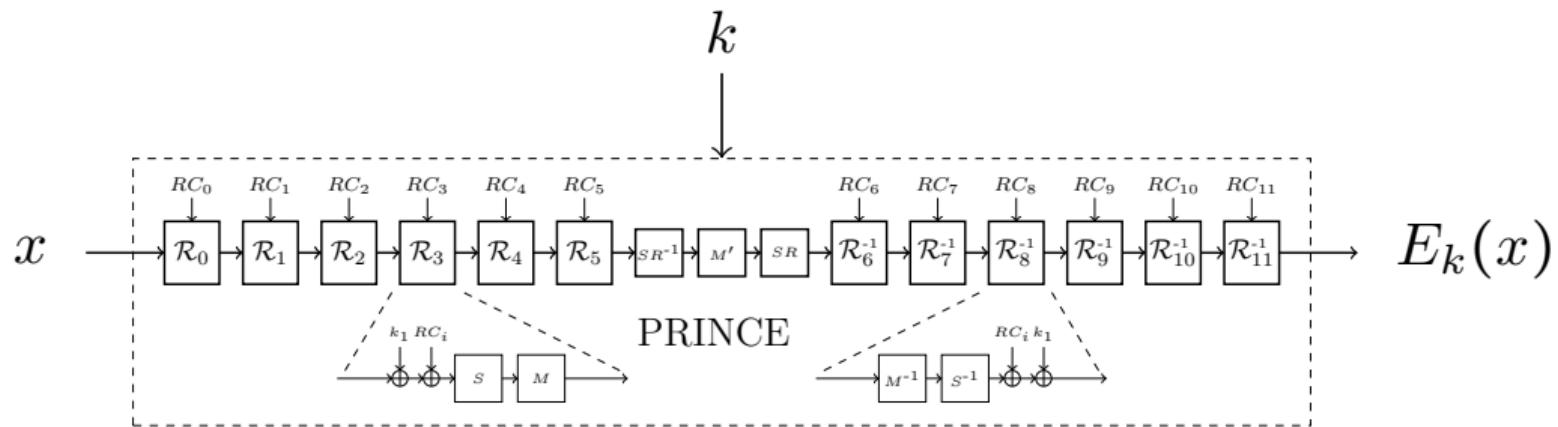


White-box model

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-
- The most **challenging** direction (this work):
white-box implementations of
existing symmetric primitives,
e.g. the AES block cipher



Example: Secure White-box



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x	$E(x)$
0000000000000000	9333dd078833edd3
0000000000000001	7072b89243c84359
0000000000000002	7838040f2b7f9af6
0000000000000003	0b502e4231f42da3
0000000000000004	c39ea8c9434252aa
...	
fffffffffffffb	8f1a82bc7af09497
fffffffffffffc	9aaf33009a8e9a2f
fffffffffffffd	5cd335922f9f0236
fffffffffffffe	39d0e8b9a0eded09
ffffffffffffff	daf2ced4ab8fc658

Example: Secure White-box

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...	
fffffffffffffb	8f1a82bc7af09497
fffffffffffffc	9aaf33009a8e9a2f
fffffffffffffd	5cd335922f9f0236
fffffffffffffe	39d0e8b9a0ed09
ffffffffffffff	daf2ced4ab8fc658

Impractical! 128 exabytes for a 64-bit cipher!

White-box: Industry vs Academia



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- WB has many applications
- strong need for *efficient* WB
- industry **does** WB:
hidden designs

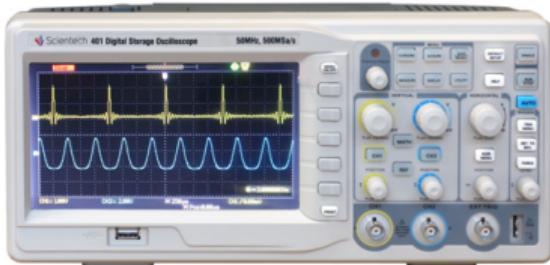
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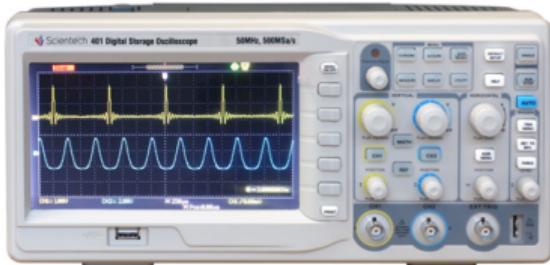
- **theory:** approaches using iO/FE, currently *impractical*
- **practical WB-AES:**
few attempts (2002-2017),
all broken
- powerful DCA attack
(CHES 2016)

White-Box: Differential Computation Analysis (DCA)



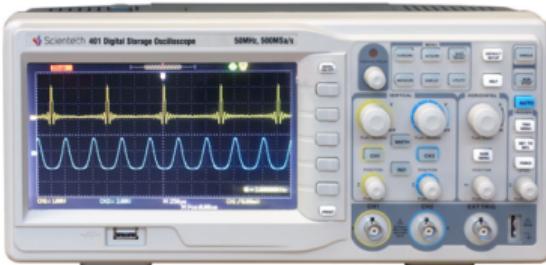
- DCA = Differential Power Analysis (DPA)
applied to white-box implementations
- Most of the implementations **broken automatically**

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- Side-channel protection: **masking schemes**

White-Box: Differential Computation Analysis (DCA)



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- Side-channel protection: **masking schemes**

this work:

Can we apply the masking protection for white-box impl.?

General Setting

- Boolean **circuits**
- **obfuscated** reference implementation

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- **predictable values**: computations from ref. impl., e.g.

$$s = \text{Bit}_1(\text{SBox}(pt_1 \oplus k_1))$$

General Setting

- Boolean **circuits**
- **obfuscated** reference implementation
- **predictable values**: computations from ref. impl., e.g.

$$s = \text{Bit}_1(SBox(pt_1 \oplus k_1))$$

- **masking**: $\exists v_1, \dots, v_t$ nodes (*shares*), $f : \mathbb{F}_2^t \rightarrow \mathbb{F}_2$ s.t. for any encryption

$$f(v_1, \dots, v_t) = s$$

Masking Schemes

- **Example** Boolean masking: linear decoder $f = \bigoplus_i v_i$
- **Example** FHE: complex non-linear decoder f

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- Aim for **efficient** schemes: relatively small t (number of shares)

⇒ can be secure only if
the locations of the shares in the circuit are unknown!

this work: exploring this possibility

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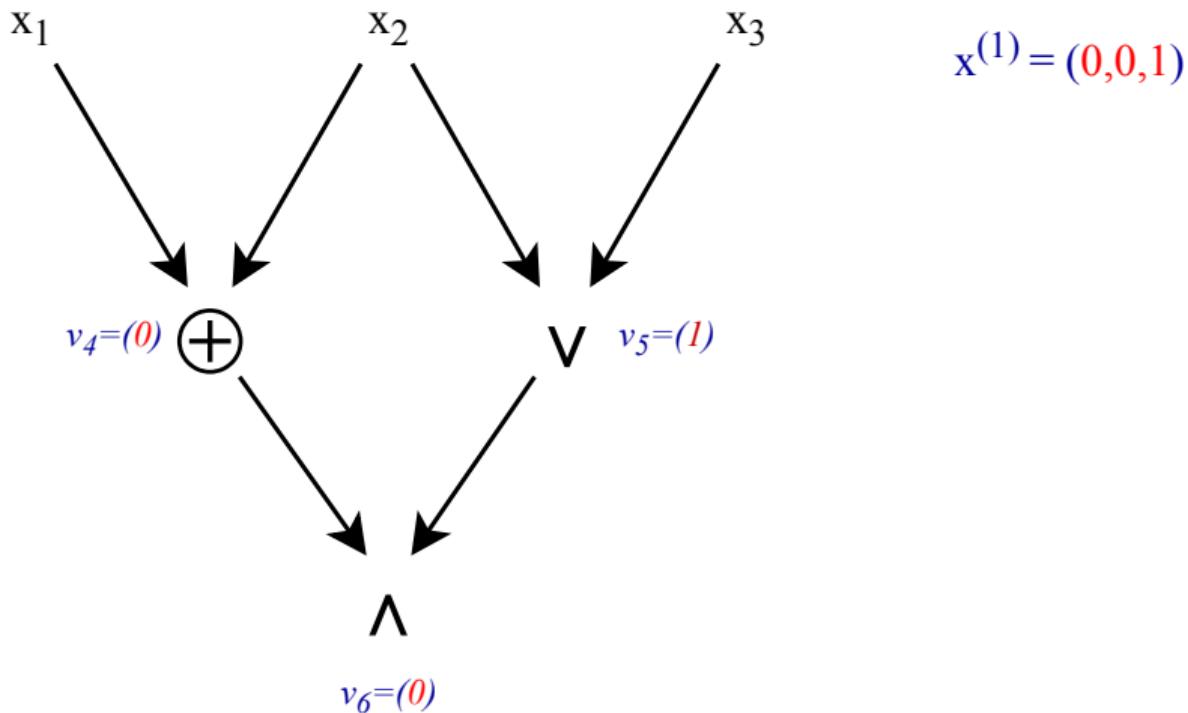
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(Generalized) Differential Computation Analysis (DCA)

$$v_1 = (0)$$

$$v_2 = (0)$$

$$v_3 = (1)$$

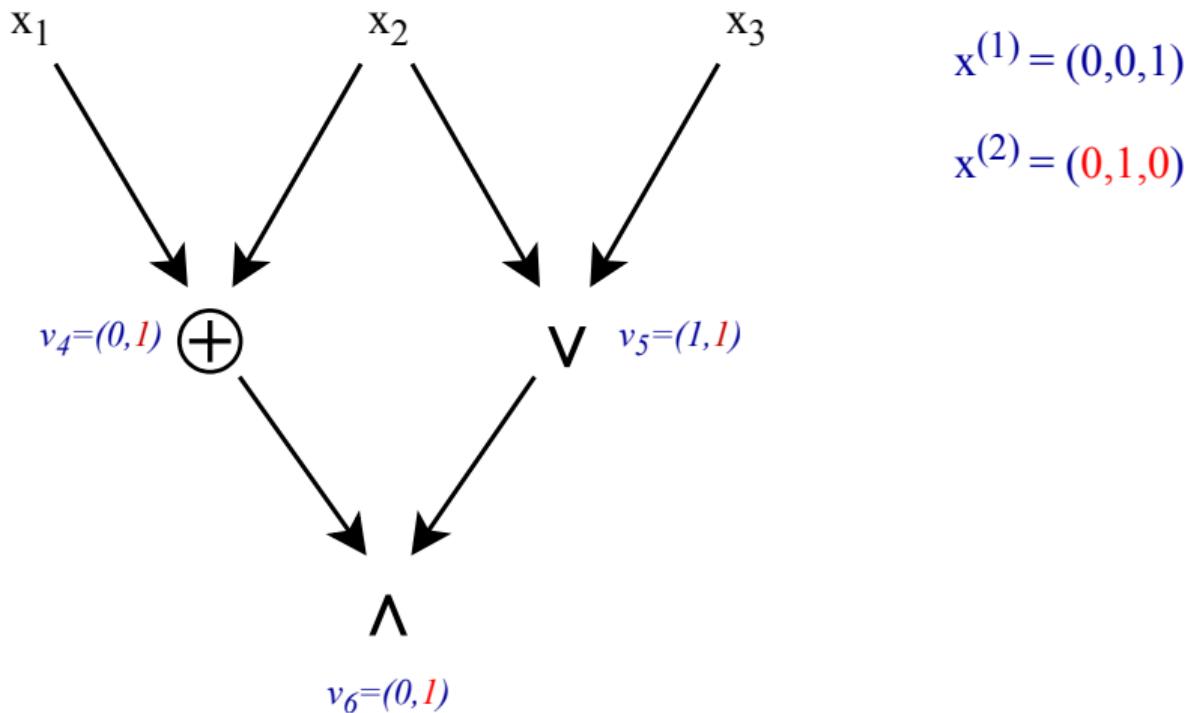


(Generalized) Differential Computation Analysis (DCA)

$$v_1 = (0, \textcolor{red}{0})$$

$$v_2 = (0, \textcolor{red}{I})$$

$$v_3 = (I, \textcolor{red}{0})$$

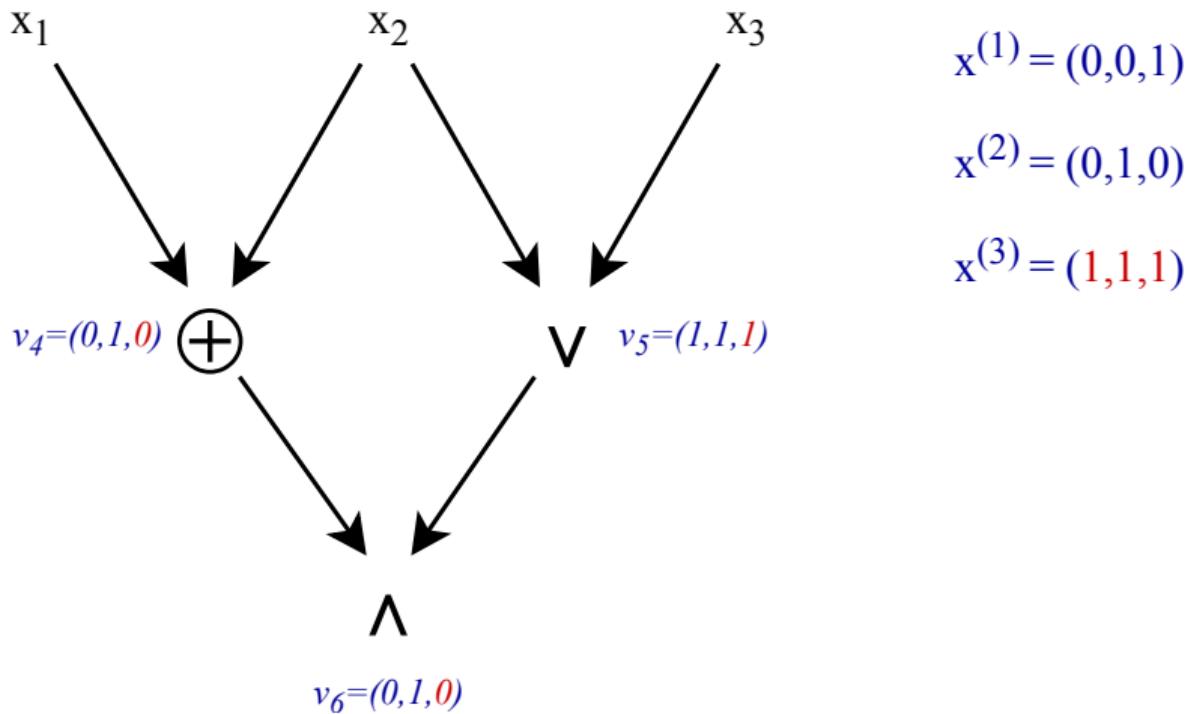


(Generalized) Differential Computation Analysis (DCA)

$$v_1 = (0, 0, \textcolor{red}{I})$$

$$v_2 = (0, 1, \textcolor{red}{I})$$

$$v_3 = (1, 0, \textcolor{red}{I})$$



The Linear Algebra Attack

- consider Boolean masking (**linear** decoder)
- matching with a **predictable value s :**
a basic linear algebra problem:

$$M \times z = s, \quad M = [v_1 \mid \dots \mid v_n]$$

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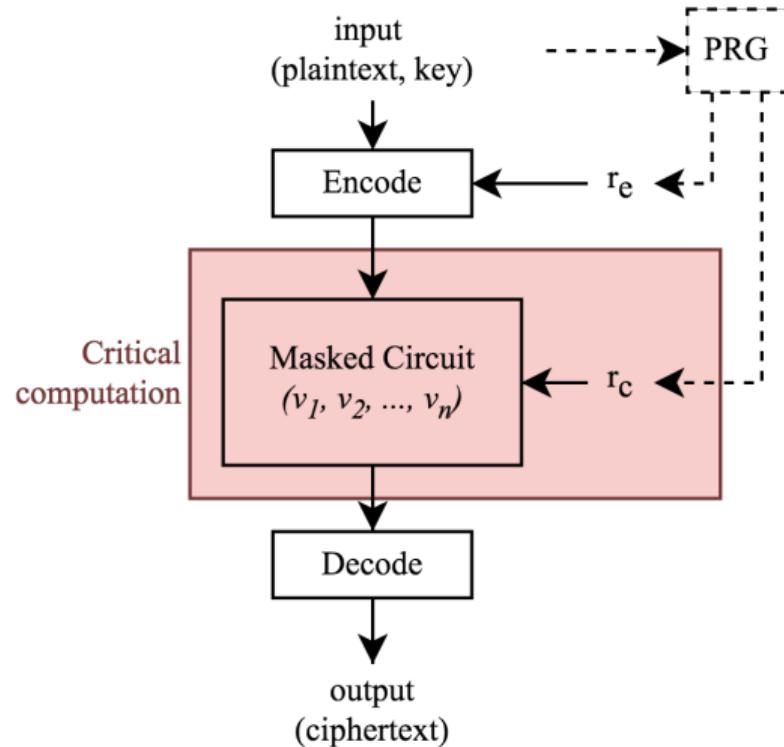
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Algebraic Security (1/3)

Security Model:

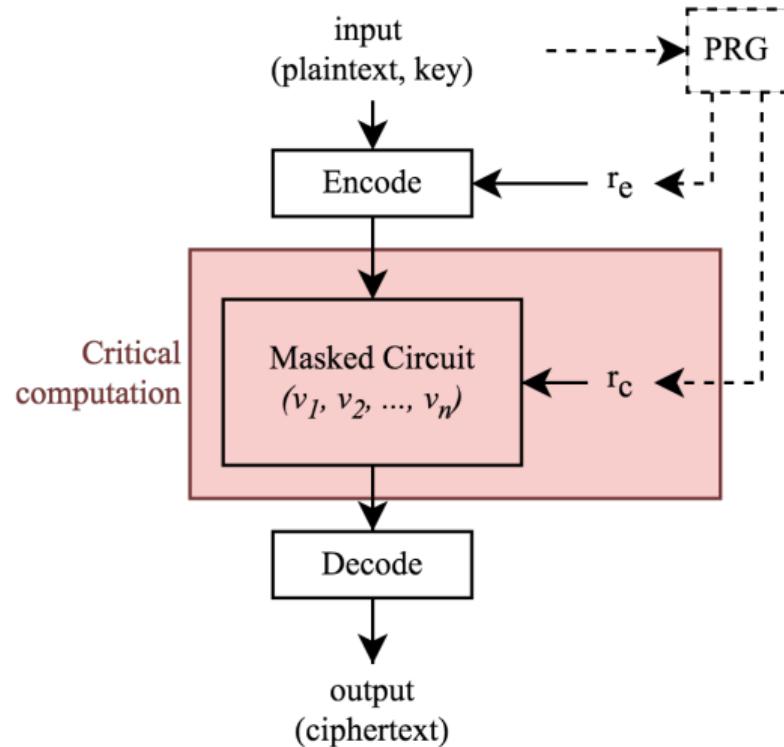
- ① **random** bits allowed
 - ▶ as in classic masking
 - ▶ model **unpredictability**
 - ▶ in WB impl. as **pseudorandom**



Algebraic Security (1/3)

Security Model:

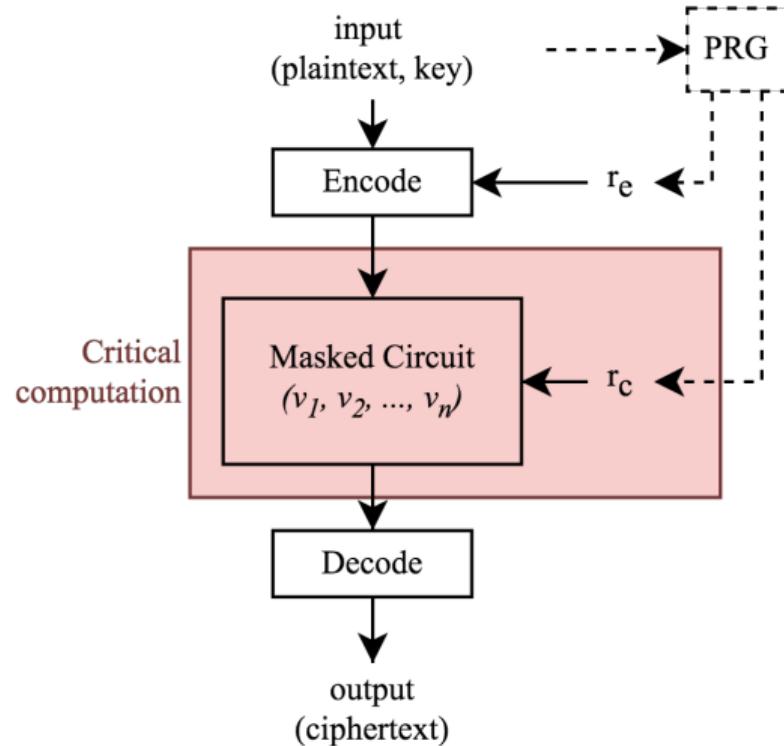
- ① **random** bits allowed
 - ▶ as in classic masking
 - ▶ model **unpredictability**
 - ▶ in WB impl. as **pseudorandom**
- ② **Goal:**
any $f \in \text{span}\{v_i\}$ is **unpredictable**



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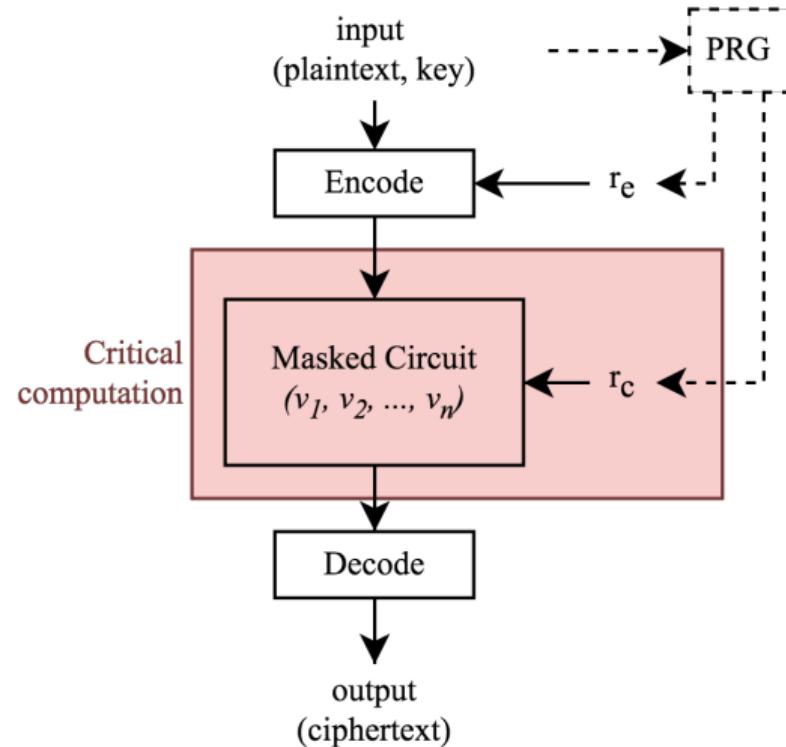
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- ② **Goal:**
any $f \in \text{span}\{v_i\}$ is **unpredictable**
- ③ **isolated** from obfuscation problems



Algebraic Security (2/3)

Adversary:

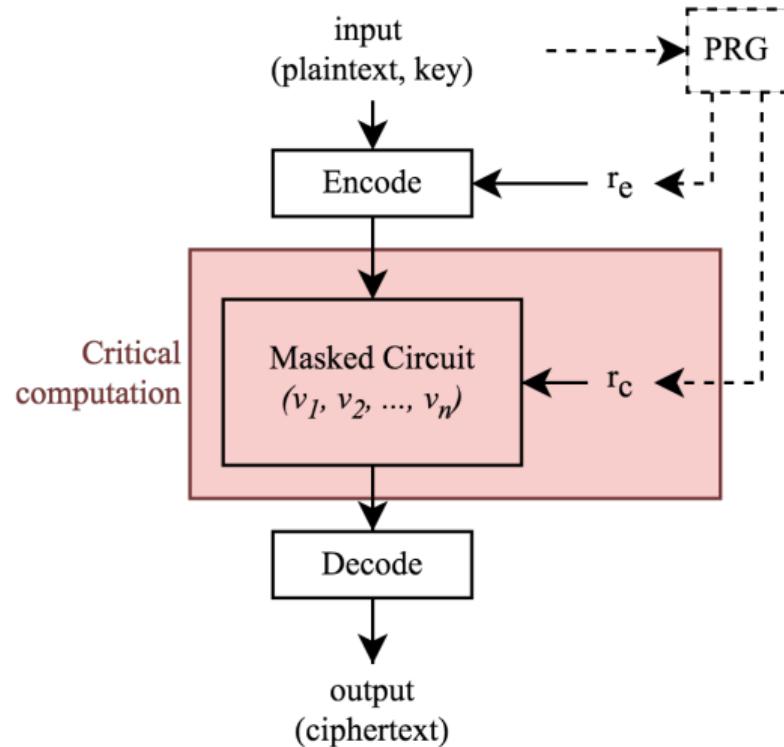
- ① chooses plaintext/key pairs



Algebraic Security (2/3)

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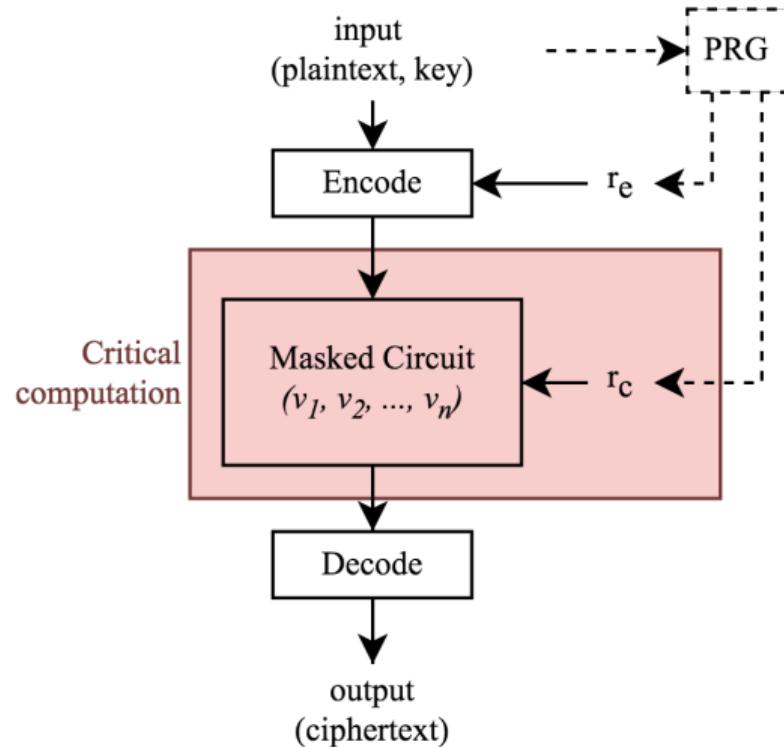
- ① chooses plaintext/key pairs
- ② chooses $f \in \text{span}\{v_i\}$



Algebraic Security (2/3)

Adversary:

- ① chooses plaintext/key pairs
- ② chooses $f \in \text{span}\{v_i\}$
- ③ tries to **predict** values of this function
(i.e. before random bits are sampled)



Algebraic Security (3/3)

Proposition

Let $\mathcal{F} = \{f(x, \cdot, \cdot) \mid f(x, r_e, r_c) \in \text{span}\{v_i\}, x \in \mathbb{F}_2^N\}$.

Let $e = -\log_2(1/2 + \max_{f \in \mathcal{F}} \text{bias}(f))$.

Then for any adversary \mathcal{A} choosing Q inputs

$$\text{Adv}[\mathcal{A}] \leq \min(2^{Q-|r_c|}, 2^{-eQ}).$$

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Corollary

Let k be a positive integer. Then for any adversary \mathcal{A}

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Information-theoretic security!

Minimalist Quadratic Masking Scheme

Masking scheme

- quadratic decoder:
 $(a, b, c) \mapsto ab \oplus c$
- set of gadgets
- provably secure composition

```
function EvalXOR((a, b, c), (d, e, f), (ra, rb, rc), (rd, re, rf))
    (a, b, c) ← Refresh((a, b, c), (ra, rb, rc))
    (d, e, f) ← Refresh((d, e, f), (rd, re, rf))
    x ← a ⊕ d
    y ← b ⊕ e
    z ← c ⊕ f ⊕ ae ⊕ bd
    return (x, y, z)

function EvalAND((a, b, c), (d, e, f), (ra, rb, rc), (rd, re, rf))
    (a, b, c) ← Refresh((a, b, c), (ra, rb, rc))
    (d, e, f) ← Refresh((d, e, f), (rd, re, rf))
    ma ← bf ⊕ rce
    md ← ce ⊕ rfb
    x ← ae ⊕ rf
    y ← bd ⊕ rc
    z ← ama ⊕ dmd ⊕ rcrf ⊕ cf
    return (x, y, z)

function Refresh((a, b, c), (ra, rb, rc))
    ma ← ra · (b ⊕ rc)
    mb ← rb · (a ⊕ rc)
    rc ← ma ⊕ mb ⊕ (ra ⊕ rc)(rb ⊕ rc) ⊕ rc
    a ← a ⊕ ra
    b ← b ⊕ rb
    c ← c ⊕ rc
    return (a, b, c)
```

Minimalist Quadratic Masking Scheme

Security

- ① algorithm to verify
that bias $\neq 1/2$
- ② max. degree on r : 4

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```

Minimalist Quadratic Masking Scheme

Security

- ① algorithm to verify
that bias $\neq 1/2$
- ② max. degree on r : 4

$$\Rightarrow \text{bias} \leq 7/16$$

for 80-bit security
we need $|r_c| \geq 940$

```
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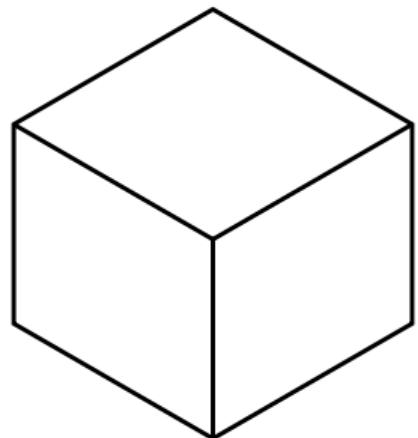
Proof-of-concept masked AES-128

- ① MQMS + 1-st order Boolean masking
- ② $31,783 \rightarrow 2,588,743$ gates expansion (x81)
- ③ 16 Mb code / 1 Kb RAM / 0.05s per block on a laptop
- ④ (unoptimized)

github.com/cryptolu/whitebox

Conclusions

- ① new attack methods \Rightarrow new **constraints** on a white-box impl.
- ② new results on **provable security** for white-box model
- ③ new links with **side-channel** research



Design and Cryptanalysis of Symmetric-Key Algorithms in Black and White-box Models

Aleksei Udovenko
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- Design of Symmetric-key Algorithms
- Structural and Decomposition Cryptanalysis
- Nonlinear Invariant Cryptanalysis
- White-box Cryptography

