

From Gray-box to White-box Cryptography

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University of Innsbruck, May 17th 2023

SnT, University of Luxembourg



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Introduction

From Gray-box to White-box

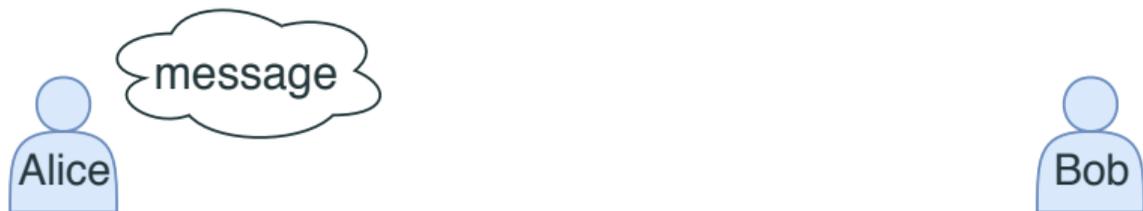
Future Research Prospects

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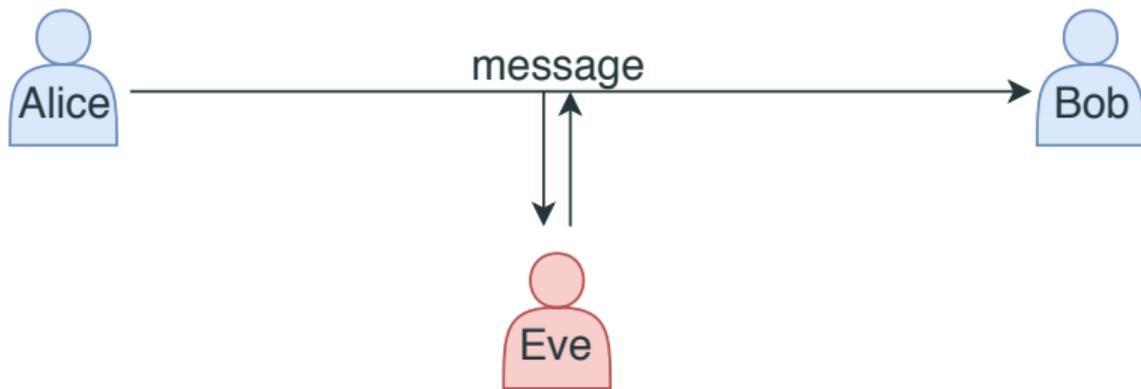
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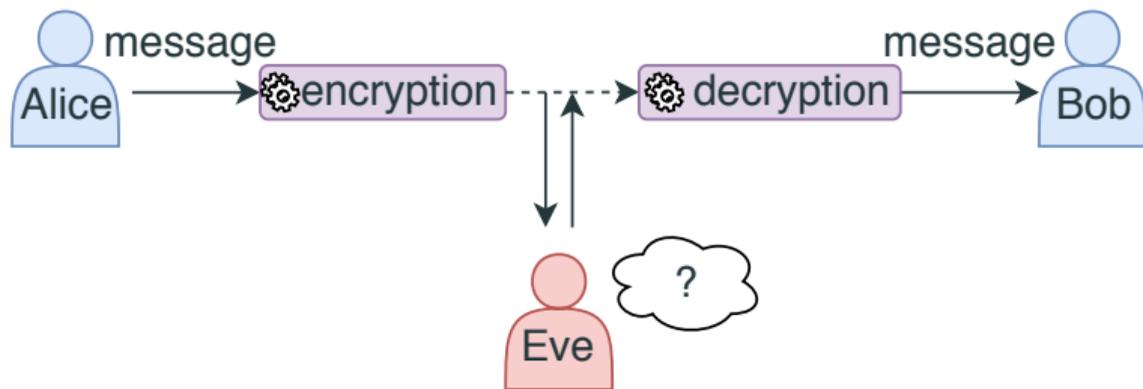
SYMMETRIC-KEY ENCRYPTION



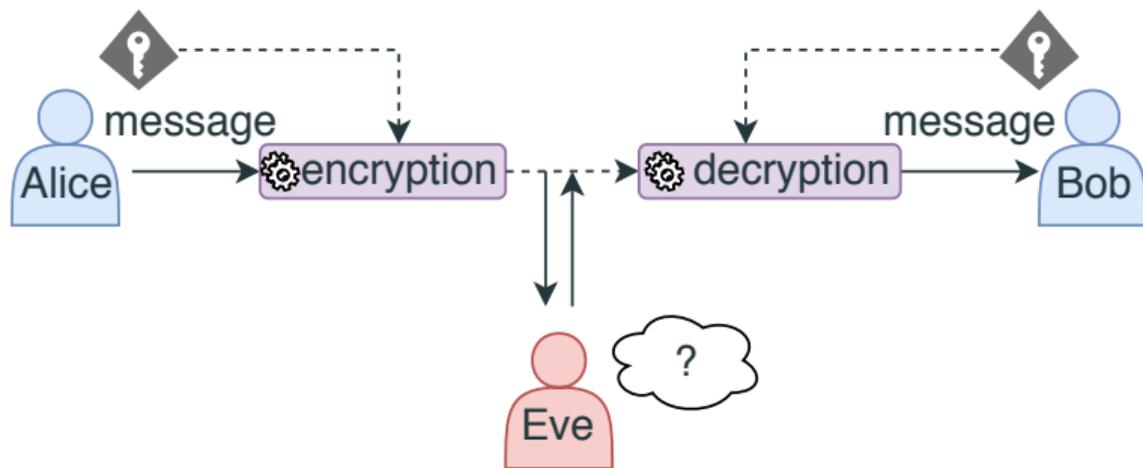
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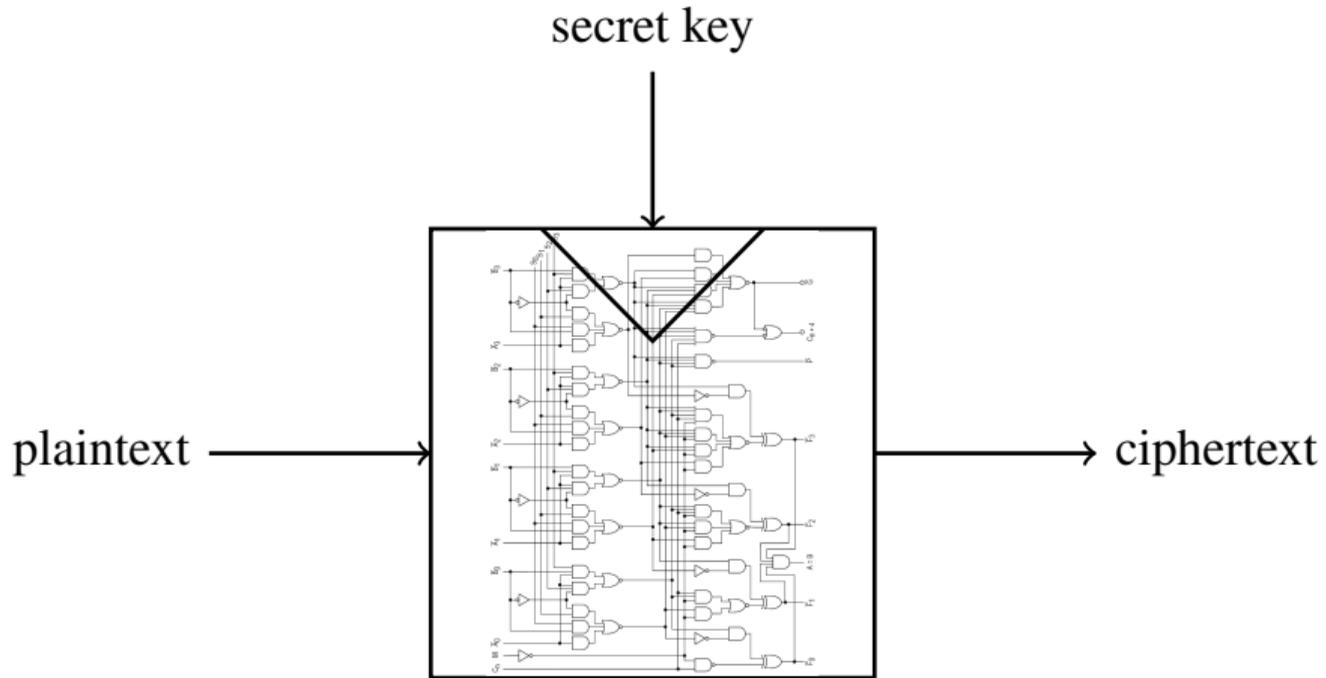
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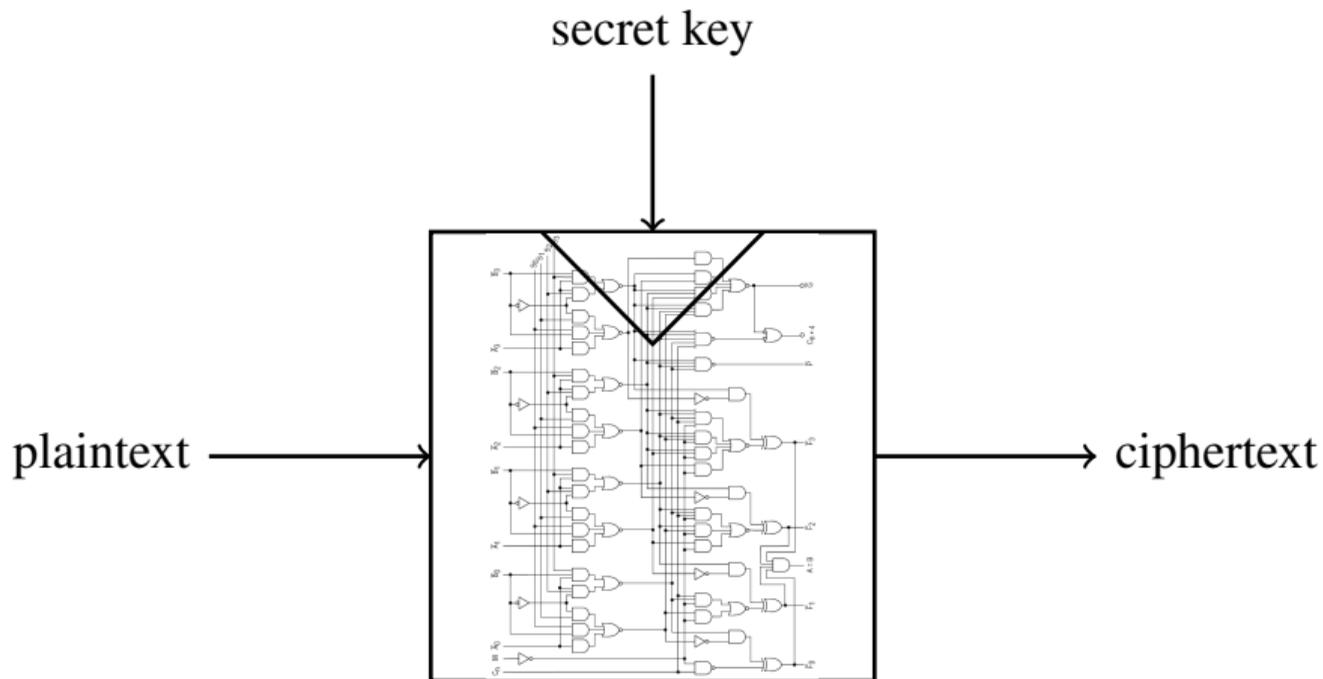
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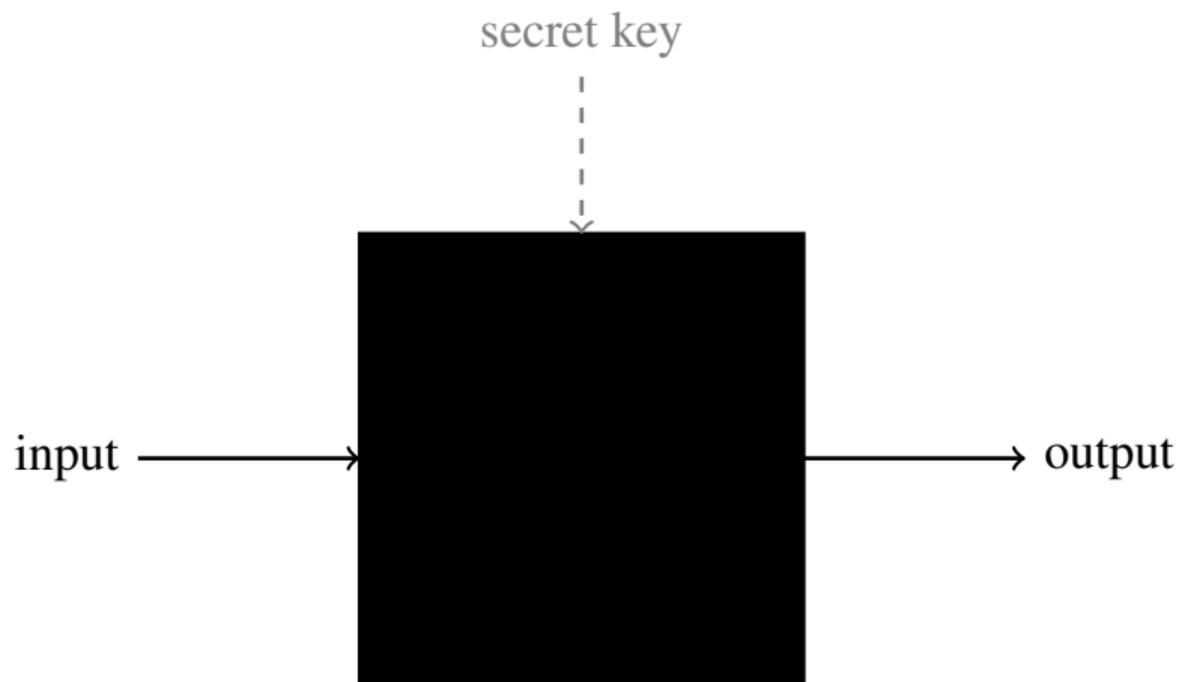
BLOCK CIPHER



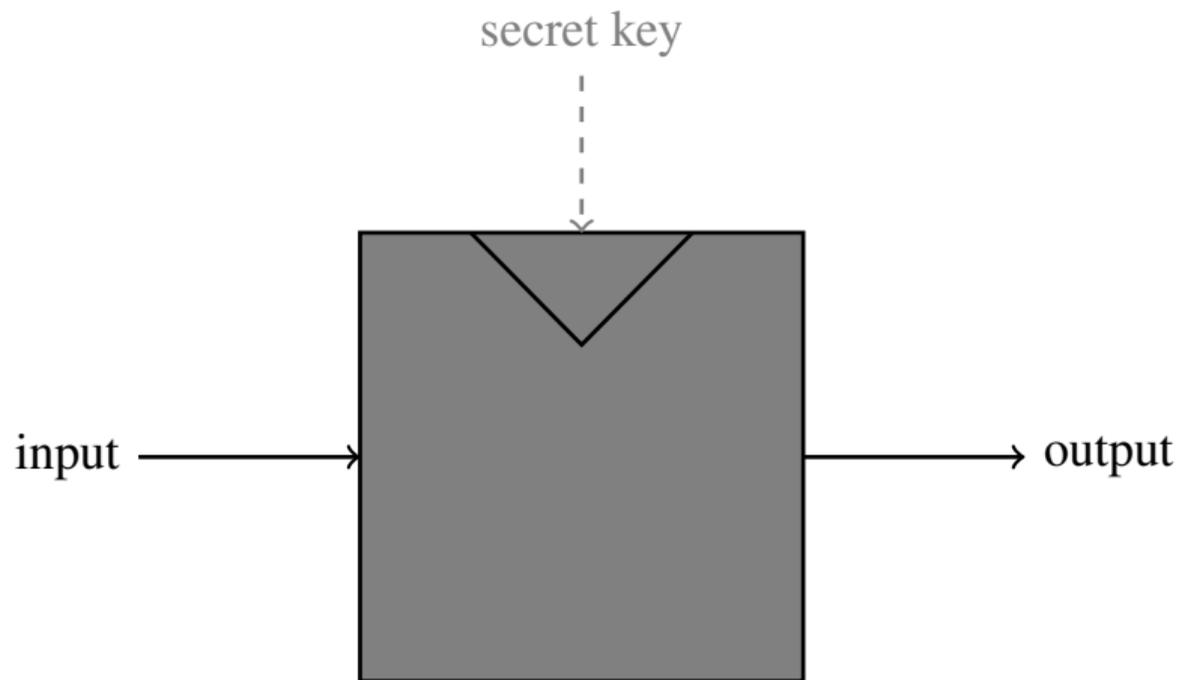
BLACK-BOX MODEL



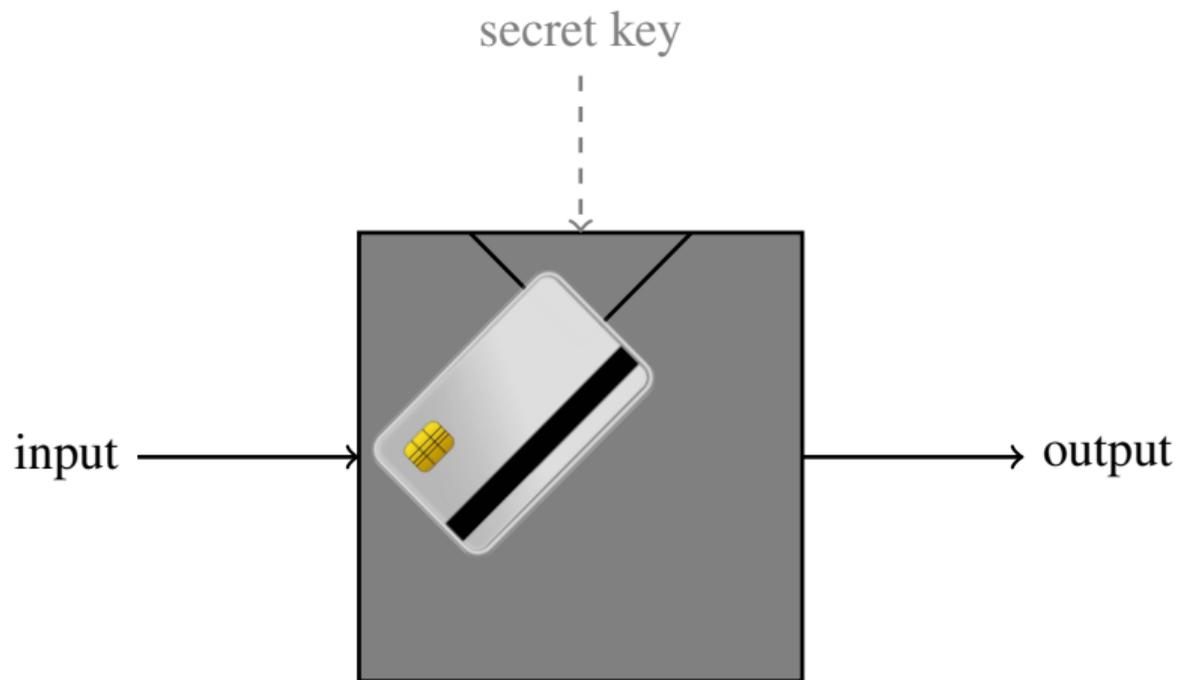
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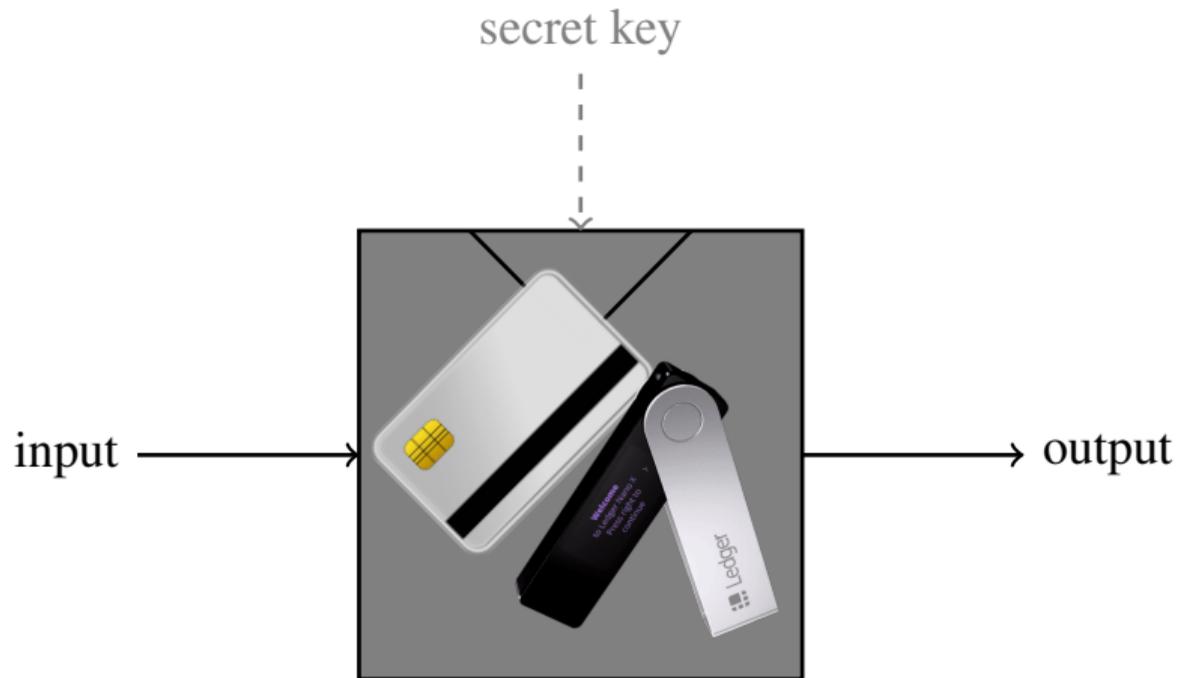
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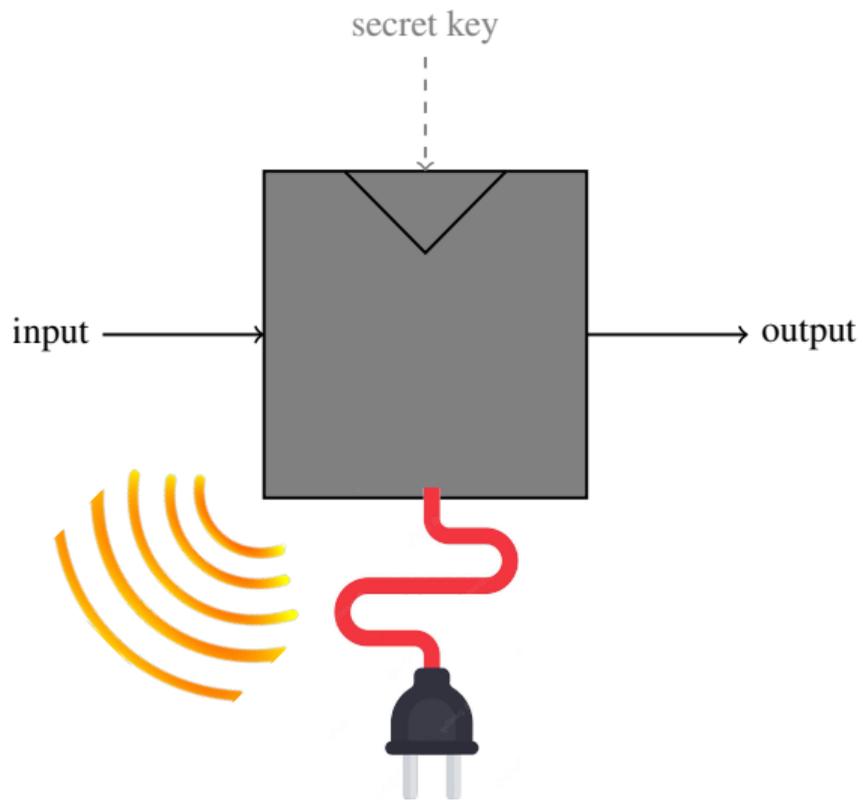
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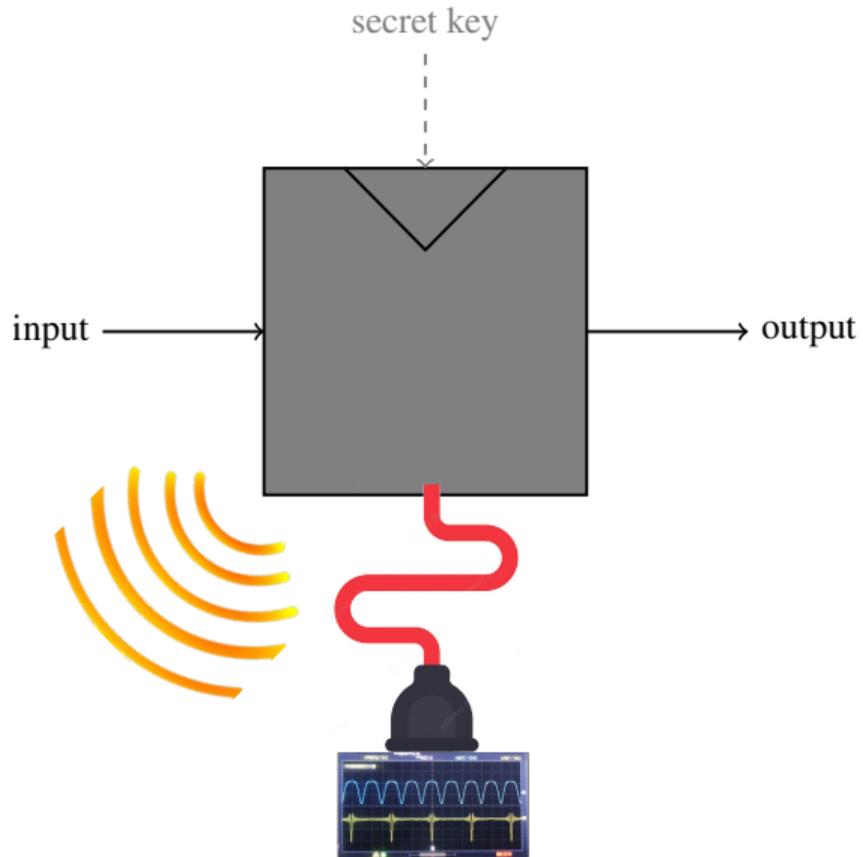
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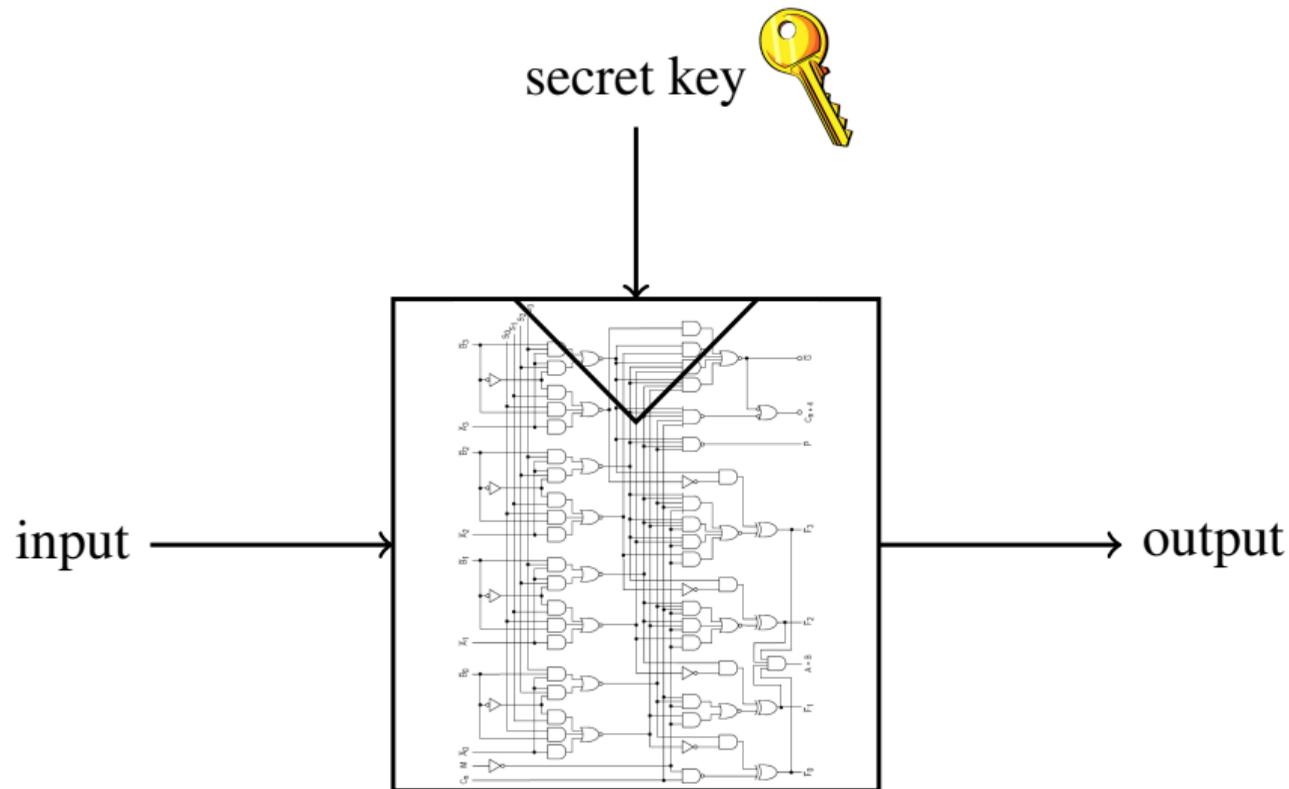


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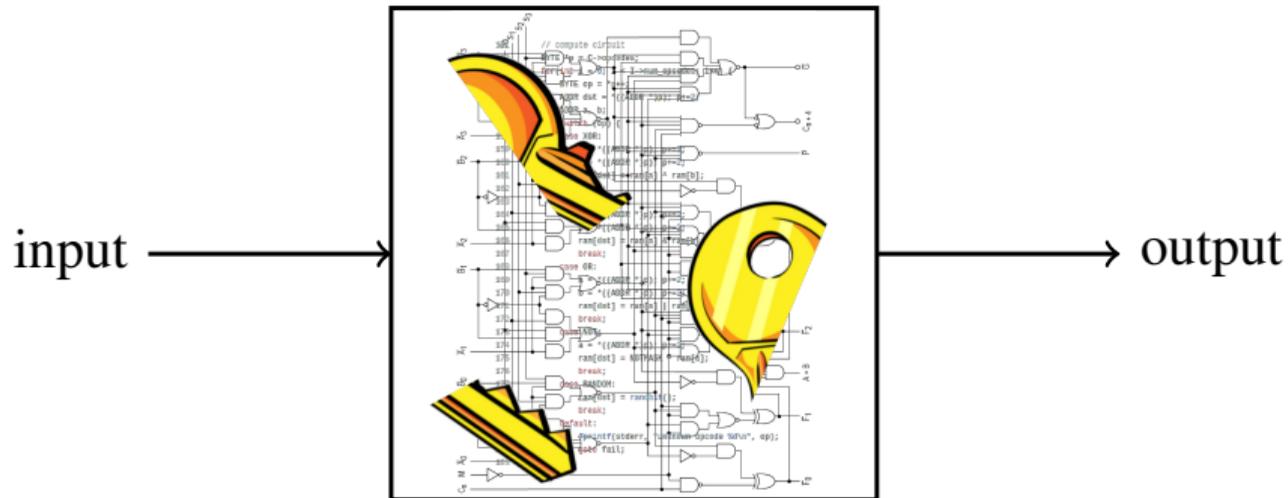


Breaking ECDSA on iPhone 4 using a \$2 probe and a sound card. [GPPTY16]

WHITE-BOX MODEL [CHOEISJOHVOOR02]

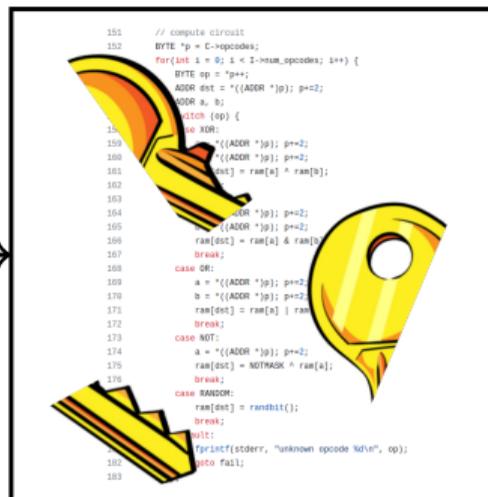


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input



output

WHY WHITE-BOX MODEL?

1. [DH76] Public-key encryption (very fast decryption)

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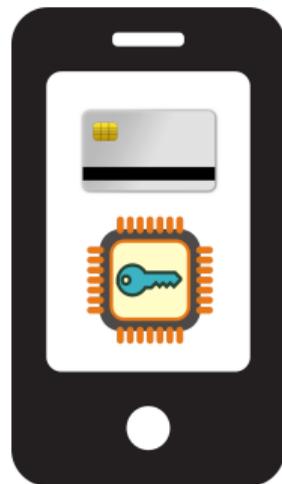
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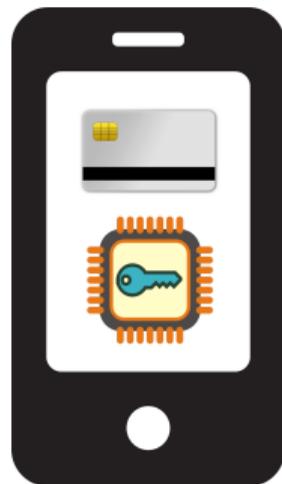
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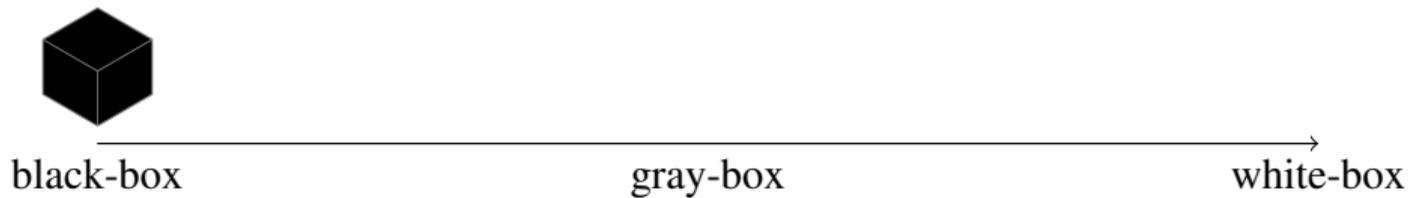


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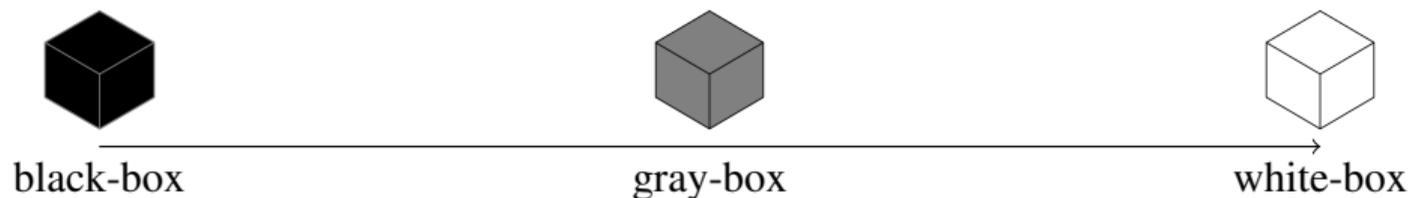
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5. General cryptographic obfuscation: advanced protocols



WHITE-BOX HISTORY OVERVIEW



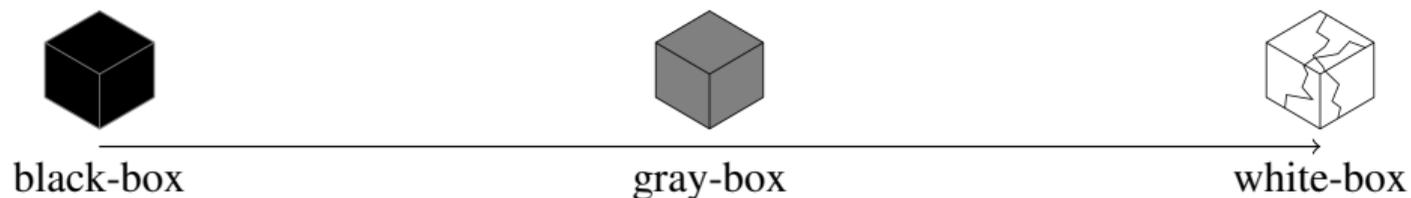
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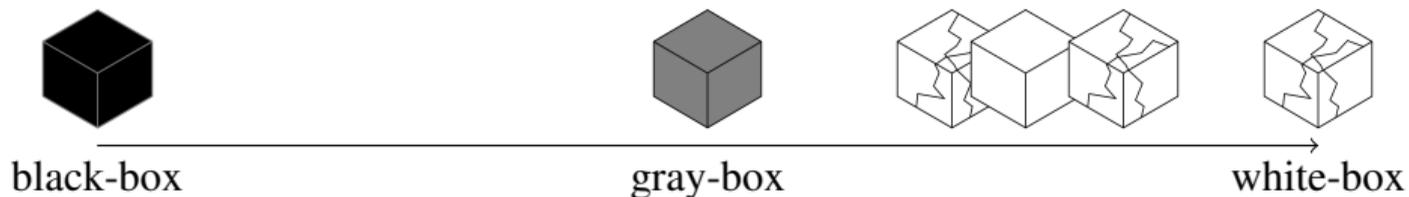


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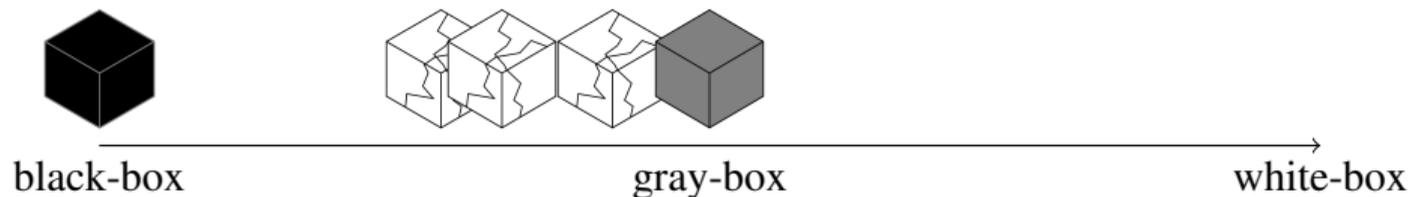
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gray-box attacks break "white-box" designs (correlation, faults)



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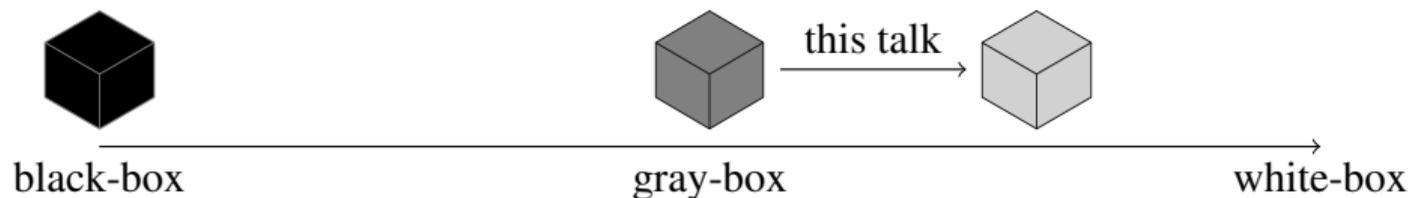
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algebraic attacks and countermeasures (this talk)



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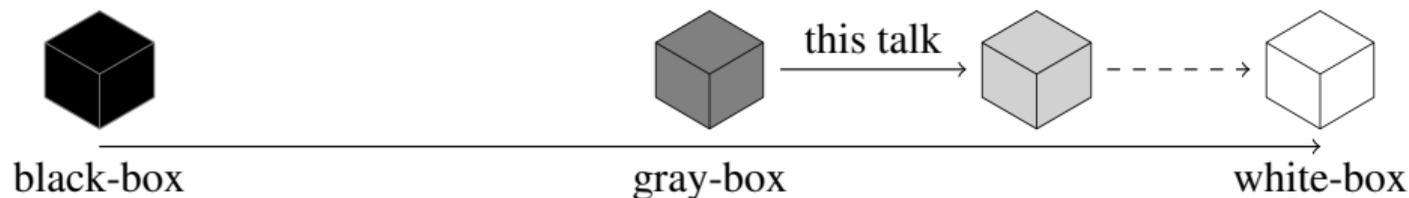
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DIFFERENTIAL POWER ANALYSIS [KOCJAFJUN99]

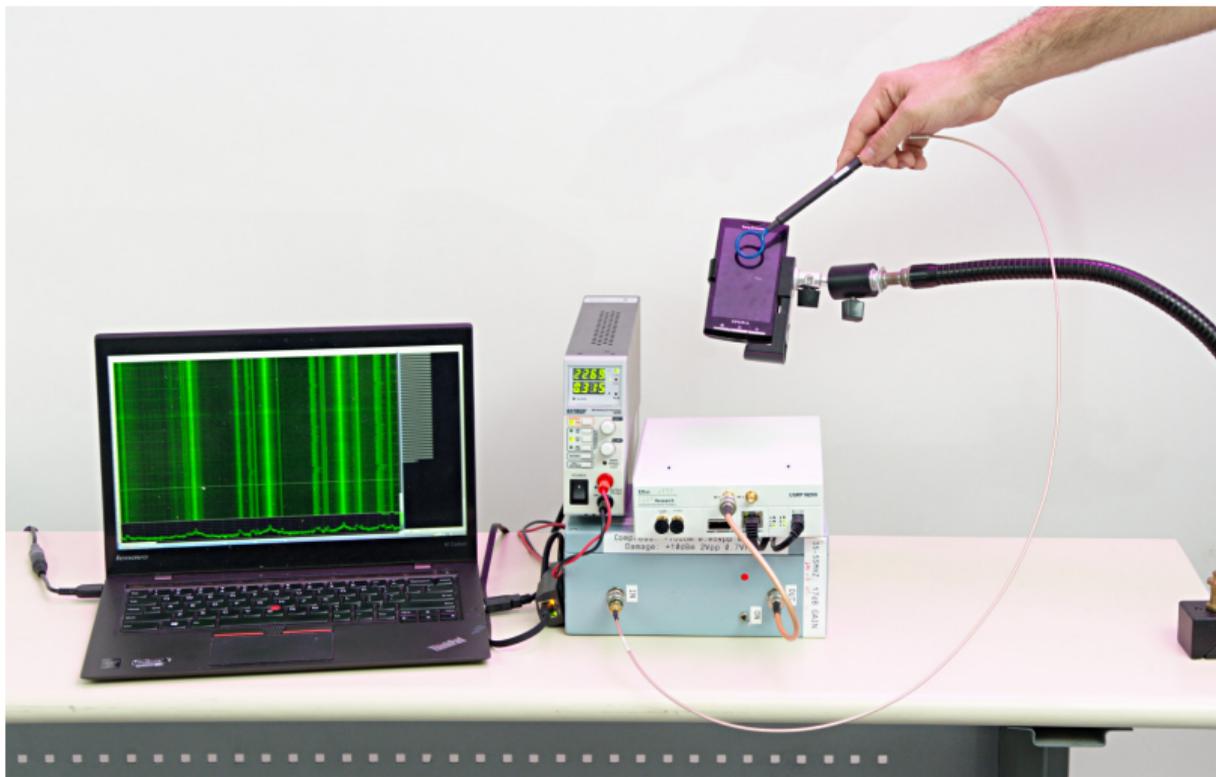
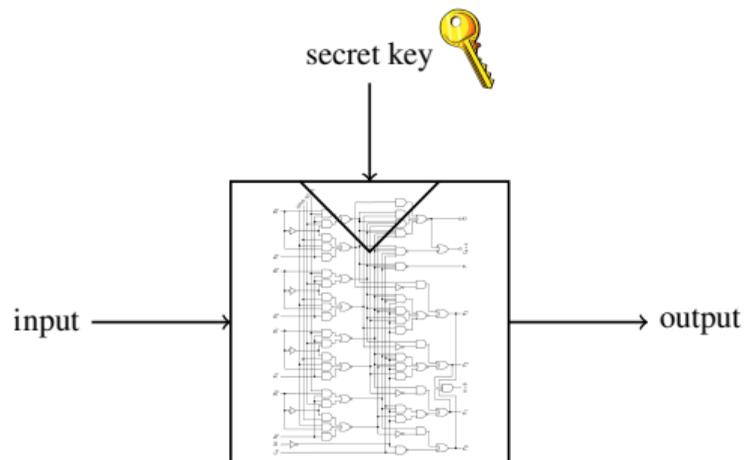
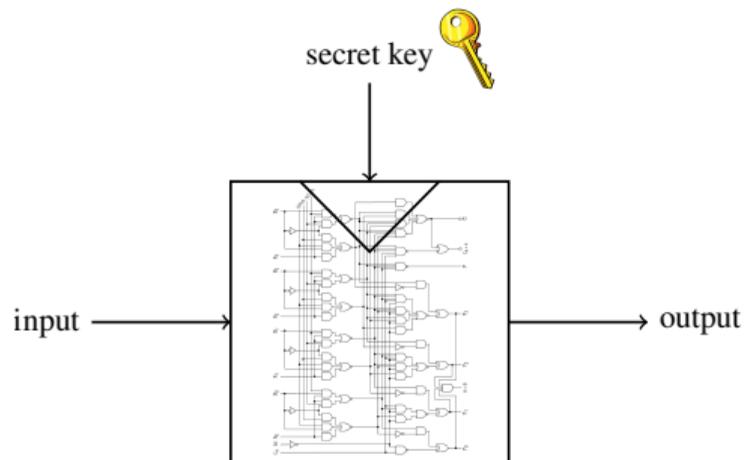


Figure: [GPPTY16] (CCS 2016)

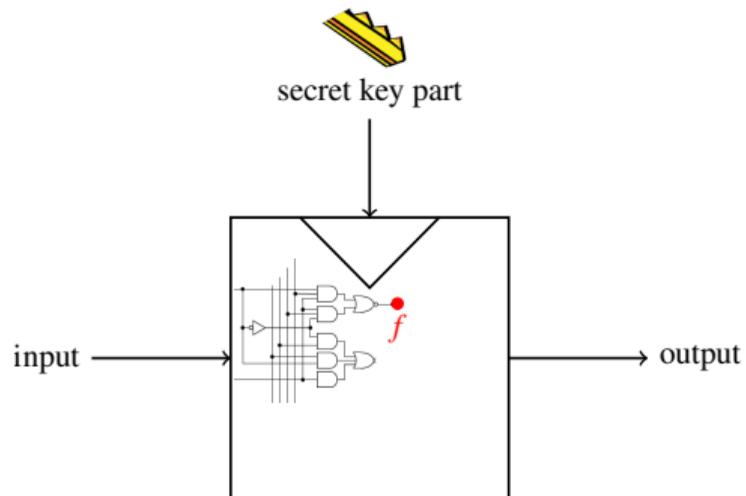
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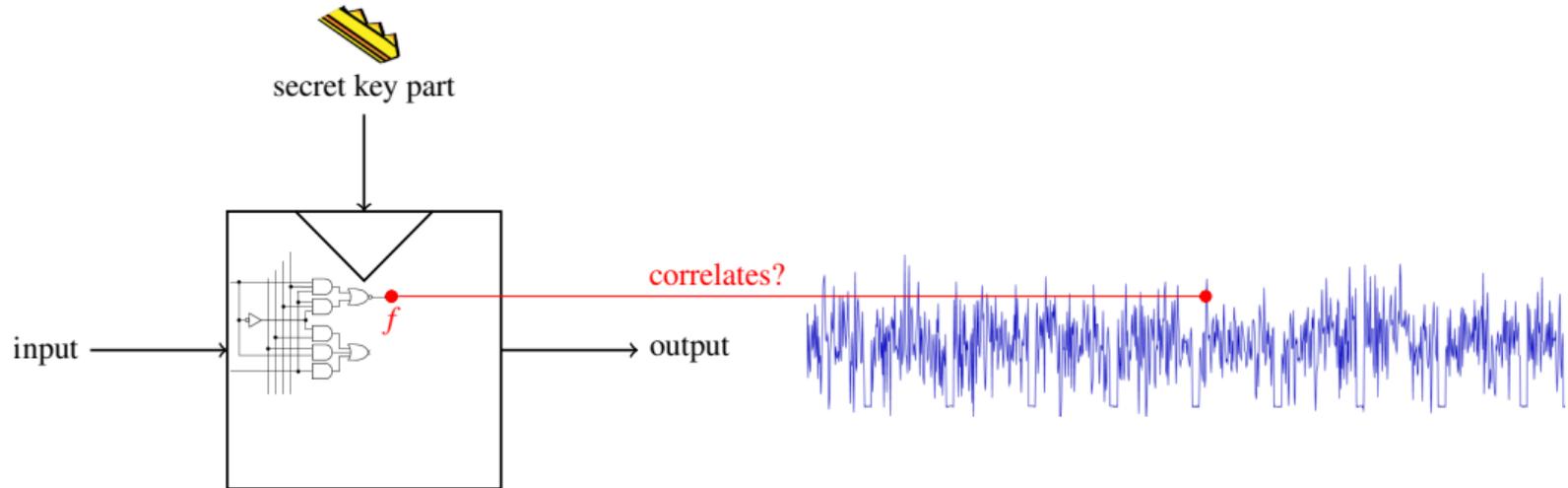
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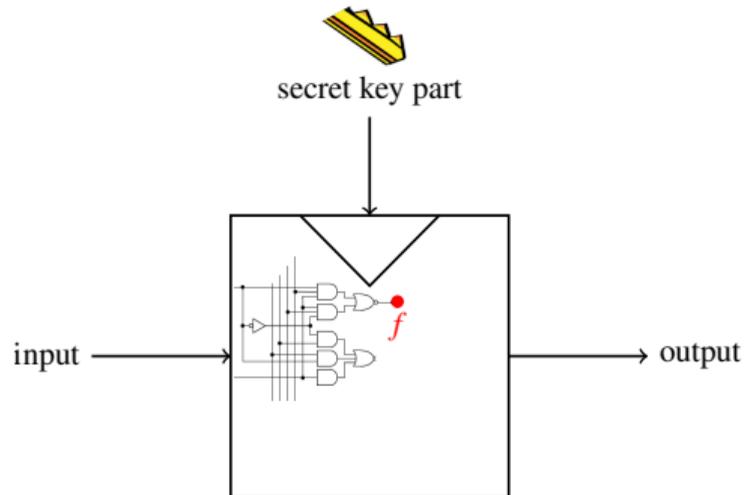
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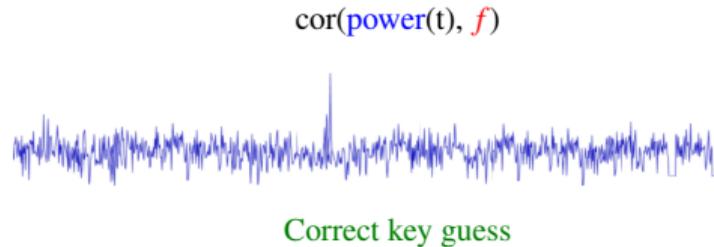
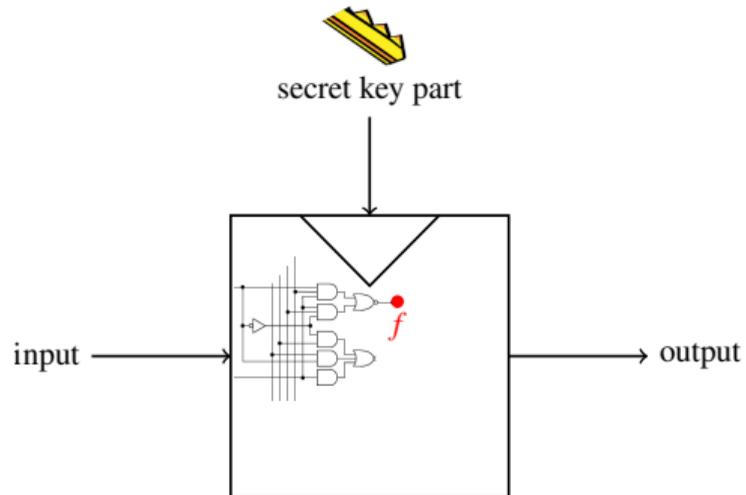


$\text{cor}(\text{power}(t), f)$

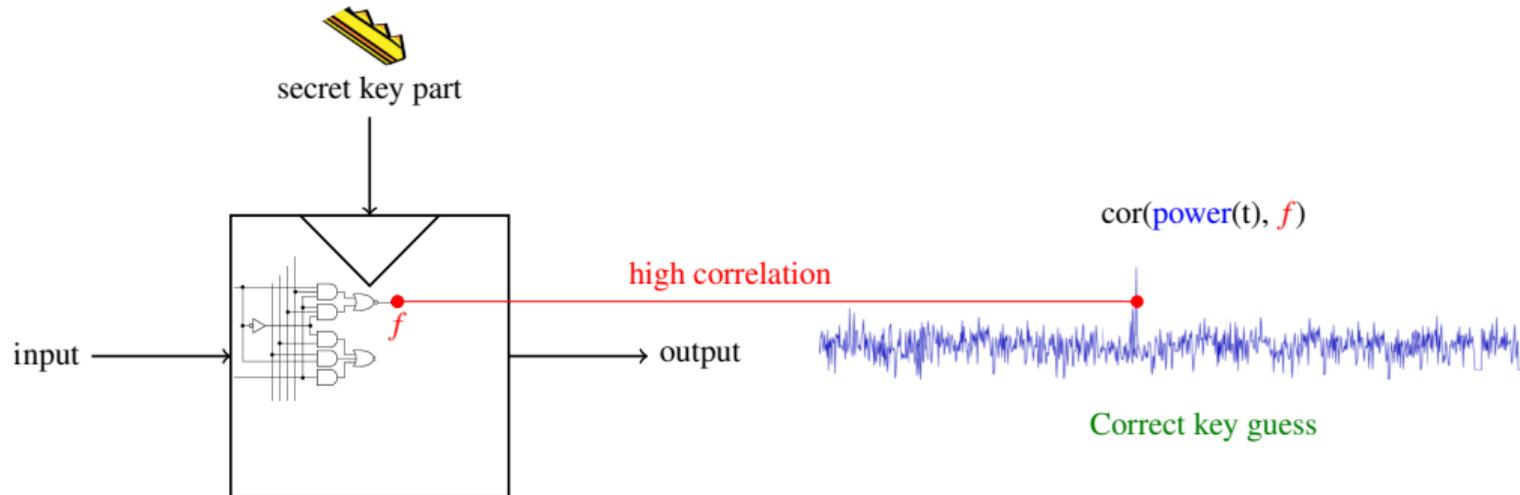


Wrong key guess

DIFFERENTIAL POWER ANALYSIS [KOCJAFJUN99]



DIFFERENTIAL POWER ANALYSIS [KOCJAFJUN99]



- Based on *Secret Sharing*
- Every intermediate value f is represented as

$$f = x_1 + \dots + x_n$$

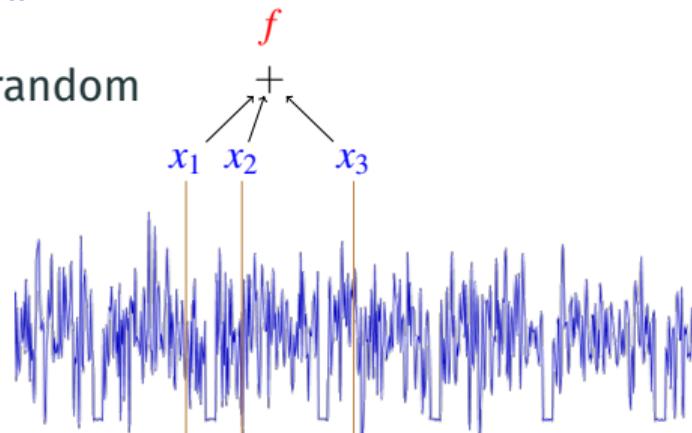
- where x_1, \dots, x_{n-1} are sampled uniformly at random
- Computations on tuples (x_1, \dots, x_n)

COUNTERMEASURE 1 - LINEAR MASKING [ISW03]

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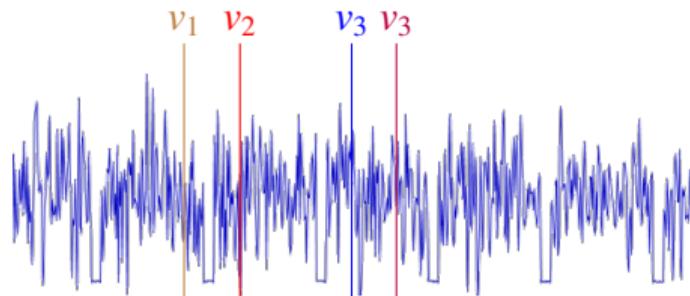


Effect 1: combinatorial explosion in locations

Effect 2: noise amplification

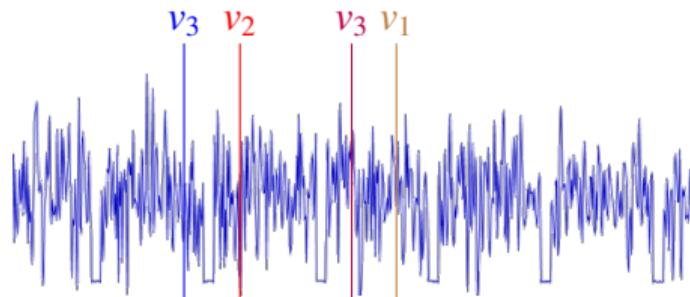
Program:

1. Operation 1
2. Operation 2
3. Operation 3
4. Operation 4



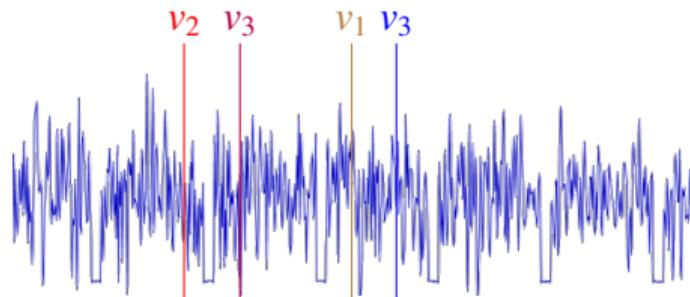
Program:

1. Operation 3
2. Operation 2
3. Operation 4
4. Operation 1



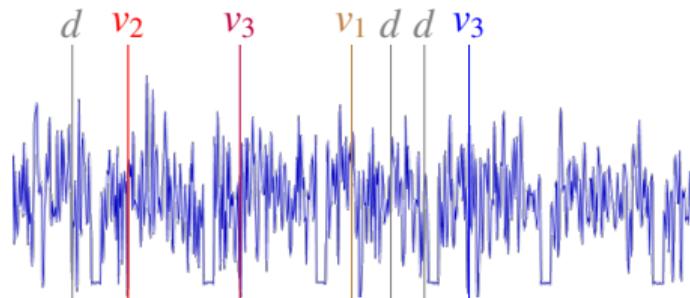
Program:

1. Operation 2
2. Operation 4
3. Operation 1
4. Operation 3



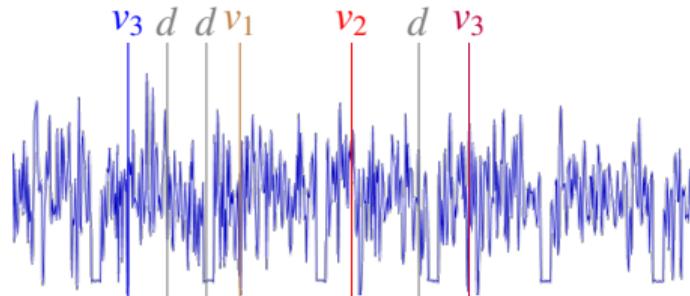
Program:

1. Dummy operation
2. Operation 2
3. Operation 4
4. Operation 1
5. Dummy operation
6. Dummy operation
7. Operation 3



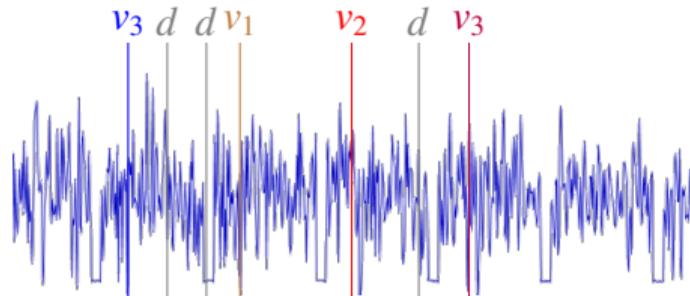
Program:

1. Operation 3
2. Dummy operation
3. Dummy operation
4. Operation 1
5. Operation 2
6. Dummy operation
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Program:

1. Operation 3
2. Dummy operation
3. Dummy operation
4. Operation 1
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- **Effect:** (small) noise amplification (aid masking)

- DPA breaks "White-box" designs [BHMT16]
- Linear masking in white-box?

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- **Problem:** no measurement noise!
- Only *locations* of shares x_1, \dots, x_n are **unknown**
- Find them using linear algebra [GPRW20; BirUdo18]

ALGEBRAIC ATTACK - ILLUSTRATION

input 1	execution traces
	010000111000

ALGEBRAIC ATTACK - ILLUSTRATION

	execution traces
input 1	010000111000
input 2	000100011101

ALGEBRAIC ATTACK - ILLUSTRATION

	execution traces
input 1	010000111000
input 2	000100011101
input 3	000101011110

ALGEBRAIC ATTACK - ILLUSTRATION

	execution traces	$f(\text{input}, \text{key})$
input 1	010000111000	1
input 2	000100011101	0
input 3	000101011110	1

ALGEBRAIC ATTACK - ILLUSTRATION

	execution traces	$f(\text{input}, \text{key})$
input 1	010000111000	1
input 2	000100011101	0
input 3	000101011110	1
...	101010111011	0
	001011100011	1
	011011011100	1
	000101100111	0
	001010001010	1
	110110101101	1
	111101100110	0
	010111111010	1
	111001110110	0
	101000000101	0
	010011100000	0
	011011000100	0
	100101010010	1

ALGEBRAIC ATTACK - ILLUSTRATION

input 1
input 2
input 3
...

execution traces

010000111000
000100011101
000101011110
101010111011
001011100011
011011011100
000101100111
001010001010
110110101101
111101100110
010111111010
111001110110
101000000101
010011100000
011011000100
100101010010

\times

$=$

$f(\text{input, key})$

1
0
1
0
1
1
0
1
1
0
1
0
0
0
0
1

ALGEBRAIC ATTACK - ILLUSTRATION

	V_1	V_2	V_3					
input 1	010000111000			\times	$=$	$f(\text{input, key})$ $= V_1 \oplus V_2 \oplus V_3$		
input 2	000100011101						0	1
input 3	000101011110						0	1
...	101010111011						0	0
	001011100011						0	1
	011011011100						1	1
	000101100111						0	0
	001010001010						0	1
	110110101101						0	1
	111101100110						1	0
	010111111010						0	1
	111001110110						0	0
	101000000101						1	0
	010011100000						0	0
	011011000100						0	0
	100101010010			0	1			

- First **security model** against algebraic attacks (gray-box style)
- First **nonlinear** quadratic scheme:

$$f = ab + c$$

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- Security proof
- Generalized and improved in [SEL21]

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- Security proof
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Algorithm 3 Minimalist Quadratic Masking Scheme.

```

1: function ENCODE( $x, r_a, r_b$ )
2:   return ( $r_a, r_b, r_a r_b \oplus x$ )

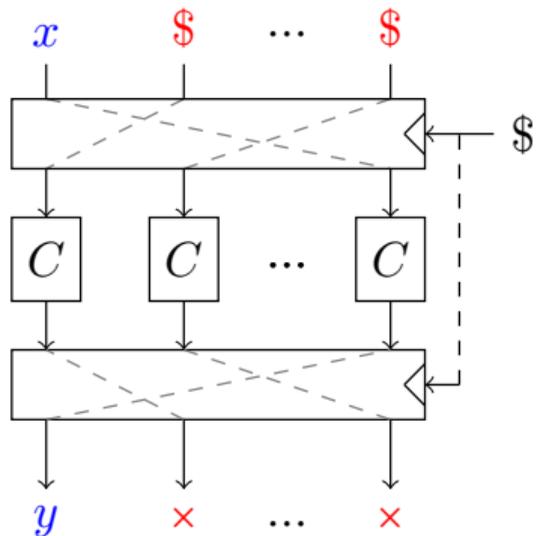
3: function DECODE( $a, b, c$ )
4:   return  $ab \oplus c$ 

5: function EVALXOR( $((a, b, c), (d, e, f), (r_a, r_b, r_c), (r_d, r_e, r_f))$ )
6:    $(a, b, c) \leftarrow \text{REFRESH}((a, b, c), (r_a, r_b, r_c))$ 
7:    $(d, e, f) \leftarrow \text{REFRESH}((d, e, f), (r_d, r_e, r_f))$ 
8:    $x \leftarrow a \oplus d$ 
9:    $y \leftarrow b \oplus e$ 
10:   $z \leftarrow c \oplus f \oplus ae \oplus bd$ 
11:  return ( $x, y, z$ )

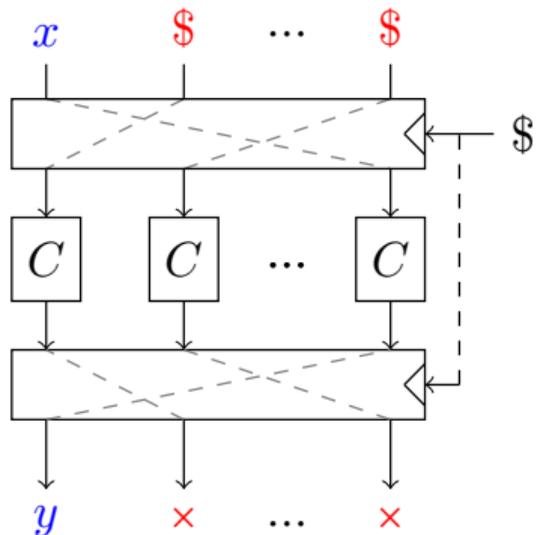
12: function EVALAND( $((a, b, c), (d, e, f), (r_a, r_b, r_c), (r_d, r_e, r_f))$ )
13:   $(a, b, c) \leftarrow \text{REFRESH}((a, b, c), (r_a, r_b, r_c))$ 
14:   $(d, e, f) \leftarrow \text{REFRESH}((d, e, f), (r_d, r_e, r_f))$ 
15:   $m_a \leftarrow bf \oplus r_c e$ 
16:   $m_d \leftarrow ce \oplus r_f b$ 
17:   $x \leftarrow ae \oplus r_f$ 
18:   $y \leftarrow bd \oplus r_c$ 
19:   $z \leftarrow am_a \oplus dm_d \oplus r_c r_f \oplus cf$ 
20:  return ( $x, y, z$ )

21: function REFRESH( $((a, b, c), (r_a, r_b, r_c))$ )
22:   $m_a \leftarrow r_a \cdot (b \oplus r_c)$ 
23:   $m_b \leftarrow r_b \cdot (a \oplus r_c)$ 
24:   $r_c \leftarrow m_a \oplus m_b \oplus (r_a \oplus r_c)(r_b \oplus r_c) \oplus r_c$ 
25:   $a \leftarrow a \oplus r_a$ 
26:   $b \leftarrow b \oplus r_b$ 
27:   $c \leftarrow c \oplus r_c$ 
28:  return ( $a, b, c$ )

```



- Extension of basic shuffling
- **Dummy** slots are essential!



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Resulting Protection:

- *Very efficient*
- Any-degree protection
- **Provably secure** (restricted)
(Boolean functions framework)

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$$f = x_1 + \dots + x_n$$

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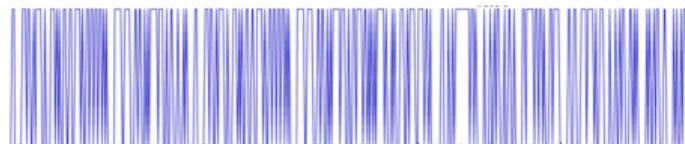
$$f = x_1 + \dots + x_n$$

3. Algebraic attack breaks linear masking (in white-box)

$$\text{TraceMatrix} \times z = f(\text{input}, \text{key})$$

SUMMARY

1. DPA breaks ad-hoc white-box designs



2. Linear masking prevents DPA

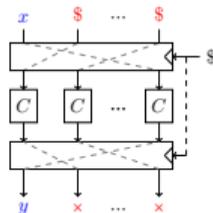
$$f = x_1 + \dots + x_n$$

3. Algebraic attack breaks linear masking (in white-box)

$$\text{TraceMatrix} \times z = f(\text{input}, \text{key})$$

4. Countermeasures: nonlinear masking and dummy shuffling

$$f = ab + c$$



WhibOx competitions 2017, 2019 (white-box AES implementations)

Pseudonym	Identities	Score
cryptolux	Alex Biryukov, Aleksei Udovenko (University of Luxembourg)	406
grothendieck	Leandro Marin (University of Murcia and Philips)	78
sebastien-riou	Sébastien Riou	66

Pseudonym	Identities	Score
cryptolux	Alex Biryukov, Aleksei Udovenko (University of Luxembourg)	3308.28
white_mountain	<i>anonymous</i>	728.22
Mugiwara	Stéphane Cauchie	666.08

Introduction

From Gray-box to White-box

Future Research Prospects

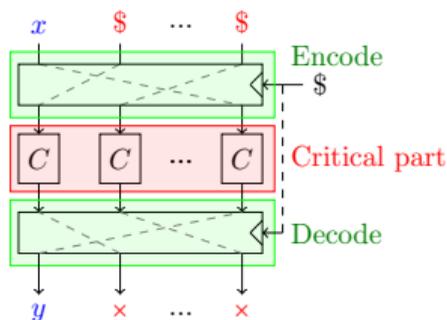
1. (Ongoing) Evaluation of mixed correlation-algebraic attacks (LPN)

$$\text{TraceMatrix} \times \mathbf{z} = f(\text{input}, \text{key}) + \text{error}$$

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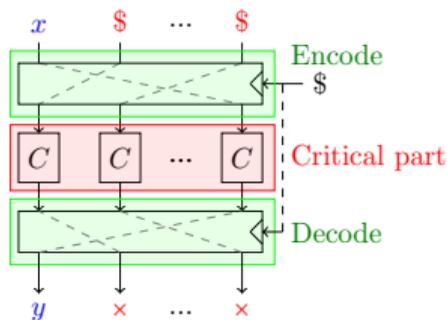
2. Extending algebraic security model to fully white-box (pseudorandomness, encoding and decoding stages)



1. (Ongoing) Evaluation of mixed correlation-algebraic attacks (LPN)

$$\text{TraceMatrix} \times \mathbf{z} = f(\text{input}, \text{key}) + \text{error}$$

2. Extending algebraic security model to fully white-box (pseudorandomness, encoding and decoding stages)



3. New nonlinear masking schemes? (degree ≥ 4 is still an open problem)

$$f = abcd + e?$$

 Fault countermeasures (white-box specifics)

 Pseudo-randomness protection

 Combination of countermeasures

 Structure-hiding methods

-  Fault countermeasures (white-box specifics)
-  Pseudo-randomness protection
-  Combination of countermeasures
-  Structure-hiding methods

Overall strategy: **provably-secure** components, *small* steps, scientific approach



White-box cryptography: 20-year-old challenging open problem



Should we try to extend the gray-box model first?

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