Algebraic Insights into the Secret Feistel Network

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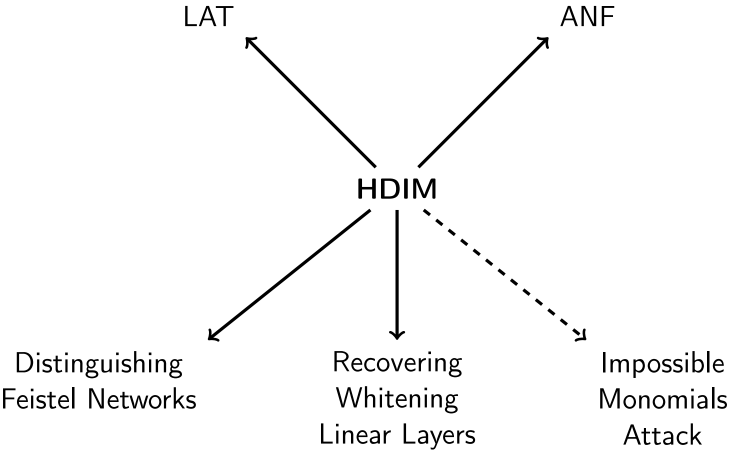
Outline


LAT
ANF
HDIM
Distinguishing Feistel Networks
Recovering Whitening Linear Layers
Impossible Monomials Attack

Plan

Linear Approximation Table (LAT)

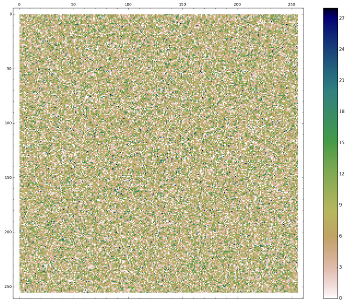
Definition (LAT, Fourier Transform, Walsh Spectrum)

The *Linear Approximation Table* of $f : \{0,1\}^n \rightarrow \{0,1\}^m$ is a $2^n \times 2^m$ matrix $\mathcal{L}$ where

$$\begin{aligned}\mathcal{L}[a,b] &= \#\{x \in \mathbb{F}_2^n, a \cdot x = b \cdot f(x)\} - 2^{n-1} \\ &= -\frac{1}{2} \sum_{x \in \mathbb{F}_2^n} (-1)^{a \cdot x \oplus b \cdot f(x)}.\end{aligned}$$

Jackson Pollock Representation of LAT

[Biryukov, Perrin CRYPTO2015]: graphical representation of LAT to reverse-engineer S-Boxes.



S-Box F of Skipjack

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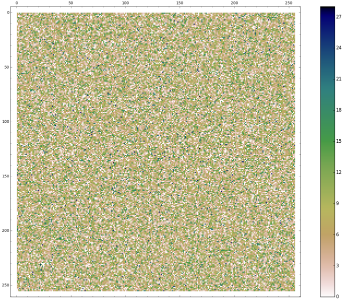

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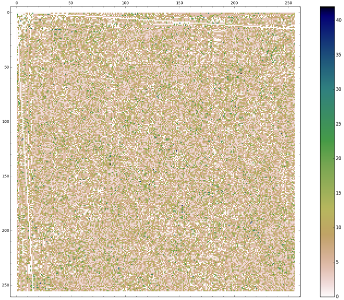

4-round Feistel Network with bijective functions

LAT modulo 4

Idea

- Look at LAT modulo 4!
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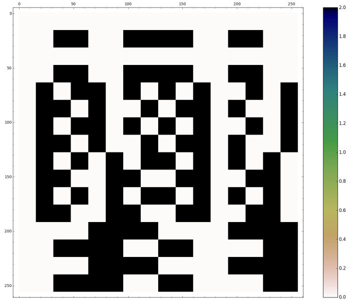

4-round Feistel Network
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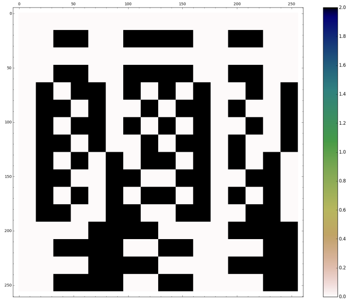

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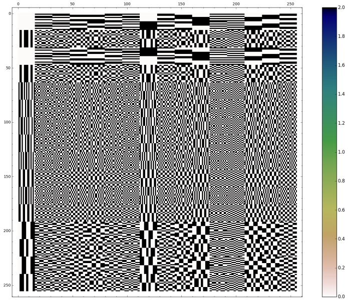

5-round Feistel Network
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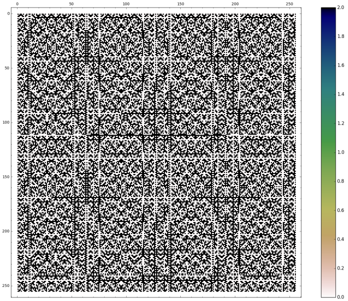

6-round Feistel Network
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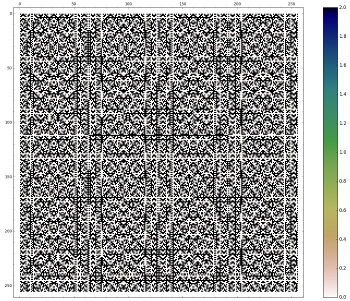

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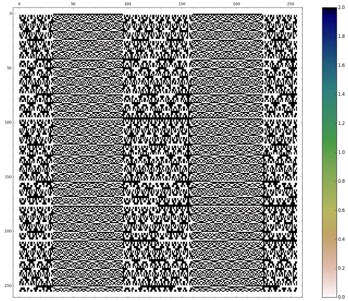

Random permutation

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- $\Rightarrow$ express $\mathcal{L}[a,b]/2$ as a vector-matrix-vector product:

$$\frac{\mathcal{L}[a,b]}{2} \equiv b^T \times \hat{H}(F) \times a \pmod 2, \tag{2}$$

where $\hat{H}(F)$ is an $n \times n$ matrix over $\mathbb{F}_2$, such that

$$\hat{H}(F)[i,j] = \bigoplus_{x \in \mathbb{F}_2^n} \big(e_i \cdot F(x)\big)\big(e_j \cdot x\big). \tag{3}$$

Another meaning of LAT modulo 4

Algebraic Normal Form (ANF)

Recall that any Boolean function f mapping n bits to 1 can be represented in a unique way as:

$$f(x) = \bigoplus_{u \in \mathbb{F}_2^n} a_u x^u = \bigoplus_{u \in \mathbb{F}_2^n} a_u \prod_{i \in [0, n-1]} x_i^{u_i}.$$

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Lemma (Another meaning of LAT modulo 4)

$\hat{H}(F)[i, j] = 1$ *if and only if the ANF of* i*th bit of* F *contains the monomial* $\prod_{k \neq j} x_k$ *(which has degree* $n - 1$*).*

High-Degree Indicator Matrix

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- For whole $\hat{H}(F)$ we need full codebook and $n2^{n-1}$ time.
- Neglible memory complexity - n bits to store the sum.

Properties of HDIM

Theorem (Linear transformations and HDIM)

Let μ, η be linear n-bit mappings, F be an n-bit permutation and let $G = \eta \circ F \circ \mu$. Then it holds that

$$\hat{H}(G) = \eta \times \hat{H}(F) \times (\mu^t)^{-1}.$$

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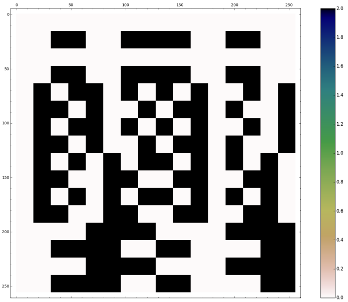
- Linear transformations applied to a permutation modify its HDIM in a linear way.
- We will use this Theorem to recover whitening linear layers.

Plan

LAT modulo 4 patterns

- Recall the LAT modulo 4 patterns that we have spotted:



4-round Feistel Network
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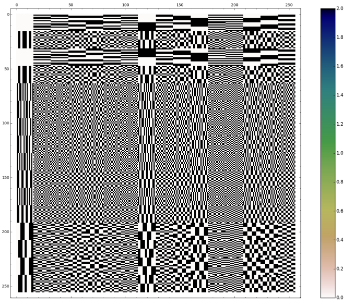

5-round Feistel Network
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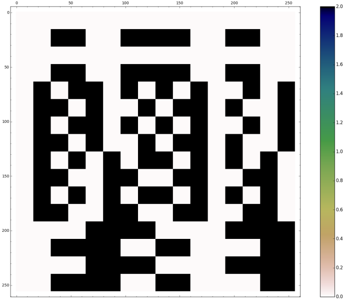
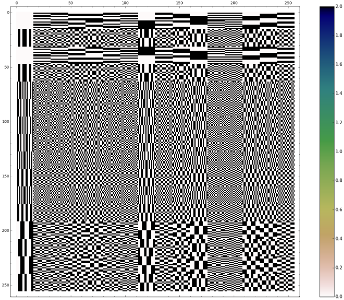
- Can be nicely rephrased in terms of **HDIM**.



4-round Feistel Network with bijective functions



5-round Feistel Network with bijective functions

HDIM Patterns in Feistel Networks

Theorem

Let F^r be r-round Feistel Network with bijective functions. Then

$$\hat{H}(\mathsf{F}^4) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & ? & ? & ? \\ 0 & 0 & 0 & ? & ? & ? \\ 0 & 0 & 0 & ? & ? & ? \end{bmatrix} \qquad \hat{H}(\mathsf{F}^5) = \begin{bmatrix} 0 & 0 & 0 & ? & ? & ? \\ 0 & 0 & 0 & ? & ? & ? \\ 0 & 0 & 0 & ? & ? & ? \\ ? & ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? & ? \end{bmatrix}$$

Example is given for $n = 3$ (6-bit Feistel Network).

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Distinguisher for Feistel Networks: one HDIM row or column is enough. Weak compared to known distinguishers for up to 5 rounds, but can attack more rounds when the degree is low.

Proof Idea

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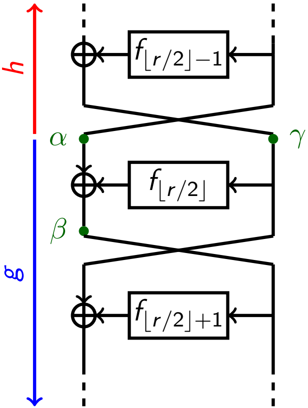

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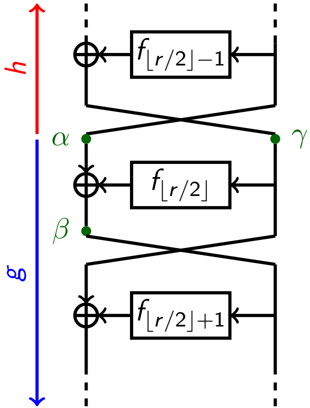

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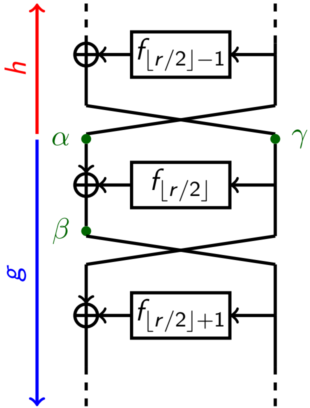
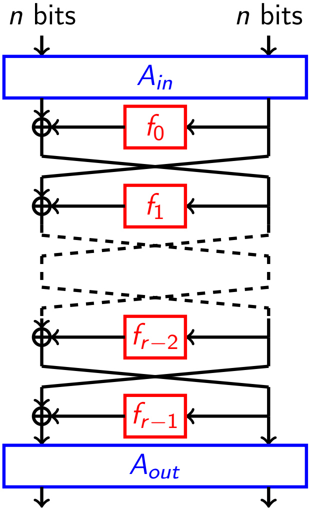
- For bijective round functions, we can get one round more by summing over α and β.



Feistel Network with Whitening Linear Layers

The AF^{r}A structure:

- Feistel Network with r rounds and n-bit branches.
- f_i: secret and independent random functions.
- whitened with secret affine layers A_{in}, A_{out}.



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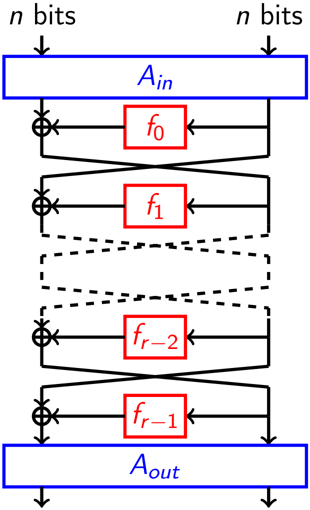
Cryptanalysis goals:

- distinguish from random permutation;
- recover the secret components.



Attacking AF^{r}A

- Let F be a Feistel Network with r rounds, such that $\hat{H}(F) = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & ? \end{bmatrix}$ (e.g. 4 rounds with bijective functions).
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- Parts of η and μ merge into the Feistel structure, so we have less unknowns and we can solve the system.
- Distinguisher for AF^{r}A and Partial recovery of linear layers.
- Complexity is dominated by computing HDIM - $n2^{2n-1}$.

Attacking one round more

- In some special cases we can attack one more round. Then we will need only that $\hat{H}(F) = \begin{bmatrix} \mathbf{0} & ? \\ ? & ? \end{bmatrix}$ (for example, 5 rounds with bijective functions).

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- Another possible case is one-sided whitening: $\mathsf{F}^{\mathsf{r}}\mathsf{A}$.
- Partial recovery of linear layers for $\mathsf{A}^{-1}\mathsf{F}^{\mathsf{r}}\mathsf{A}$ or $\mathsf{F}^{\mathsf{r}}\mathsf{A}$.
- Complexity is dominated by computing HDIM - $n2^{2n-1}$.

Plan

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- Previously, we exploited predictable absence of particular terms of degree $n-1$ in the ANFs of some output bits (entries $\hat{H}(F)_{i,j} = 0$).

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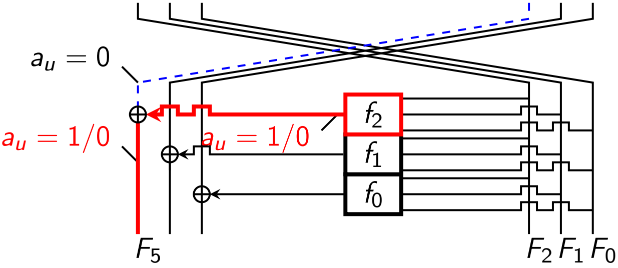
- The predictable absence of such terms may be used to recover a secret round function.

Recovery attack on 5-round Feistel Network (1/2)

- Consider a 5-round Feistel Network F with bijective round functions. Let f be the last round function.



Recovery attack on 5-round Feistel Network (1/2)

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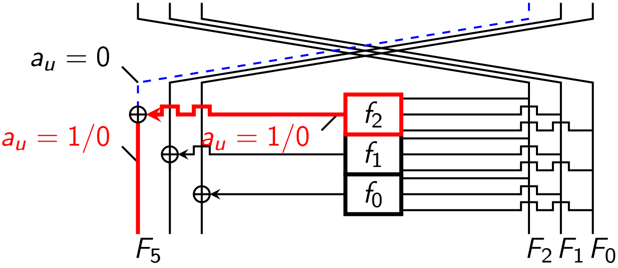
- We can prove that there are more than 2^n monomials which can't occur in the ANFs on right branch of the 4-round FN.



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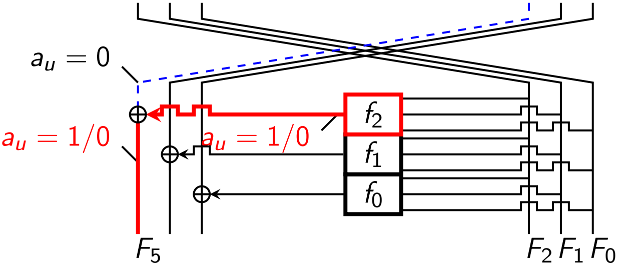
- This gives us information about the last round function f.



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- We obtain a linear system with 2^n unknowns (ANF coefficients of f_i) and more than 2^n equations.
- By solving the system we recover the secret round function f (up to a XOR constant).
- Complexity is dominated by generating the system and is $O(2^{3n})$.

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- Recovery attack when $d^{r-3} < n$. Note that the bound is not tight, the previously described attack on 5 rounds does not satisfy this condition.
- Moreover, with low degrees there are less unknowns and we need less impossible monomials.

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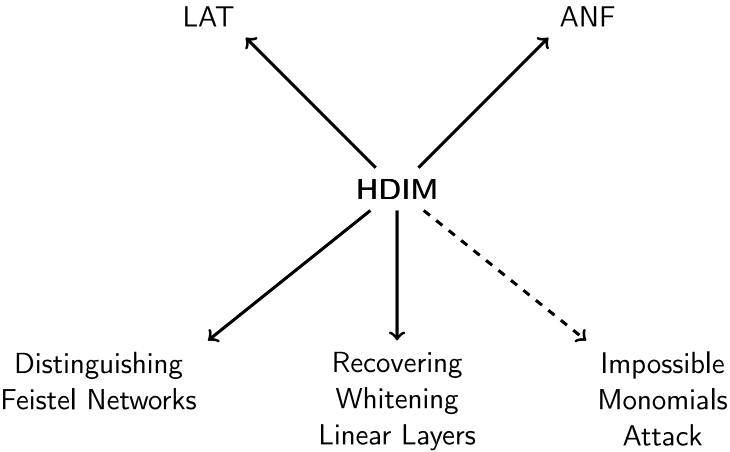
- Such cubes correspond to the absent ANF coefficients of degree $2n - 1$ (or less) which correspond to zero items in HDIM.
- The results for concrete Feistel Networks obtained by Todo are very similar to ours.

Plan

Conclusions



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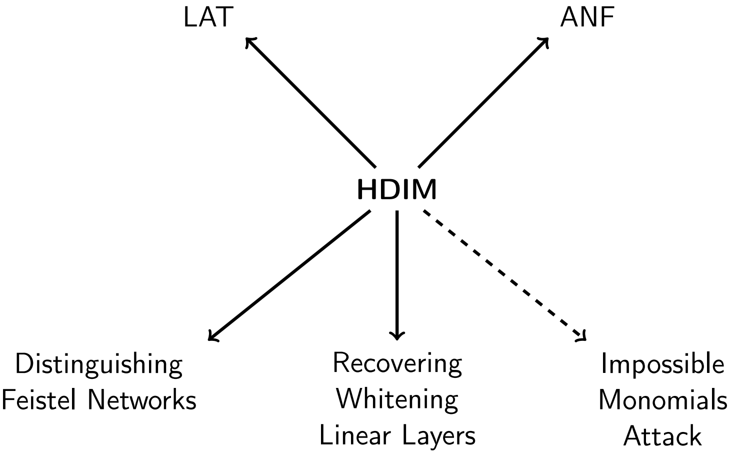

Thank you!