# Attacks and Countermeasures for White-box Designs

Alex Biryukov, Aleksei Udovenko

CSC and SnT, University of Luxembourg

December 5, 2018

UNIVERSITÉ DU LUXEMBOURG

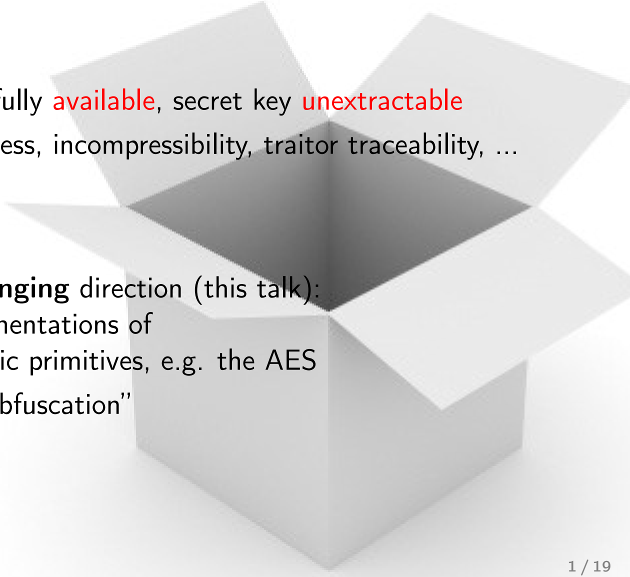
securityandtrust.lu

# Plan

# White-box

- Implementation fully available, secret key unextractable
- **Extra**: one-wayness, incompressibility, traitor traceability, ...

# White-box



- Implementation fully available, secret key unextractable
- **Extra**: one-wayness, incompressibility, traitor traceability, ...

- The most **challenging** direction (this talk):
  white-box implementations of
  existing symmetric primitives, e.g. the AES
- "Cryptographic obfuscation"

# White-box: Industry vs Academia


# White-box: Industry vs Academia

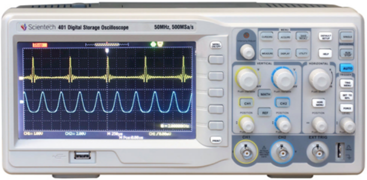

- many applications
- strong need for *practical* white-box
- industry **does** WB: hidden designs

# White-box: Industry vs Academia


- many applications
- strong need for *practical* white-box
- industry **does** WB: hidden designs

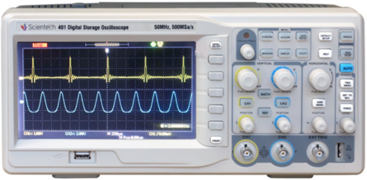
- **theory**: approaches using iO/FE, currently *impractical*
- **practical WB-AES**: few attempts (2002-2017), all broken
- powerful DCA attack (CHES 2016)

# White-Box: Differential Computation Analysis (DCA)



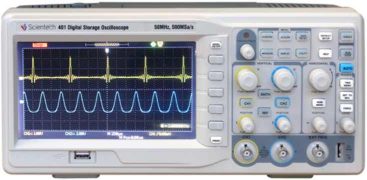

- **DCA = Differential Power Analysis (DPA)**
  applied to white-box implementations
- Most of the implementations broken automatically

# White-Box: Differential Computation Analysis (DCA)




- **DCA = Differential Power Analysis (DPA)**
  applied to white-box implementations
- Most of the implementations broken automatically
- Side-Channel protection: **masking schemes**

# White-Box: Differential Computation Analysis (DCA)




- **DCA = Differential Power Analysis (DPA)** applied to white-box implementations
- Most of the implementations broken automatically
- Side-Channel protection: **masking schemes**

*this talk*:

Can we apply the masking protection for white-box impl.?

# General Setting

- Boolean **circuits**
- **Obfuscated** reference implementation

# General Setting

- Boolean **circuits**
- **Obfuscated** reference implementation
- **Predictable values**: computations from ref. impl., e.g.

$$s = Bit_1(SBox(pt_1 \oplus k_1))$$

# General Setting

- Boolean **circuits**
- **Obfuscated** reference implementation
- **Predictable values**: computations from ref. impl., e.g.

$$s = Bit_1(SBox(pt_1 \oplus k_1))$$

- **Masking**: $\exists v_1, \ldots, v_t$ nodes (*shares*), $f : \mathbb{F}_2^t \to \mathbb{F}_2$ s.t. for any encryption

$$f(v_1, \ldots, v_t) = s$$

# Masking Schemes

- **Example:** Boolean masking: linear decoder $f = \bigoplus_i v_i$
- **Example:** FHE: non-linear decoder $f$

# Masking Schemes

- **Example:** Boolean masking: linear decoder $f = \bigoplus_i v_i$
- **Example:** FHE: non-linear decoder $f$
- Aim for efficient schemes: relatively small $t$ (number of shares)

# Masking Schemes

- **Example:** Boolean masking: linear decoder $f = \bigoplus_i v_i$
- **Example:** FHE: non-linear decoder $f$
- Aim for efficient schemes: relatively small $t$ (number of shares)

$\Rightarrow$ can be secure only if the locations of the shares in the circuit are unknown!

*this talk*: exploring this possibility

# Plan

# Attacks I

**Combinatorial attacks:**

- (partially) guess locations of the shares
- **probabilistic**: correlation with predictable values
- **exact**: time-memory trade-off

# Attacks I

**Combinatorial attacks:**

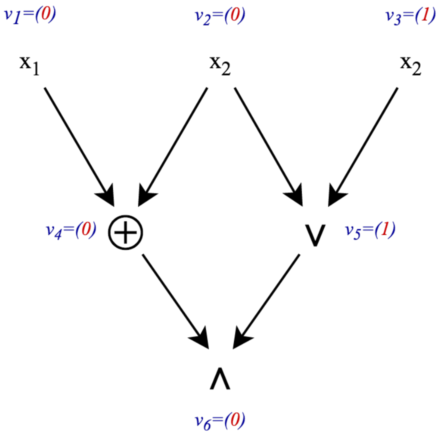
- (partially) guess locations of the shares
- **probabilistic**: correlation with predictable values
- **exact**: time-memory trade-off

**Fault attacks:**

- **new application:** recover locations of the shares
- 1- and 2- share fault injections
- applicability depends on protections

# Attacks II

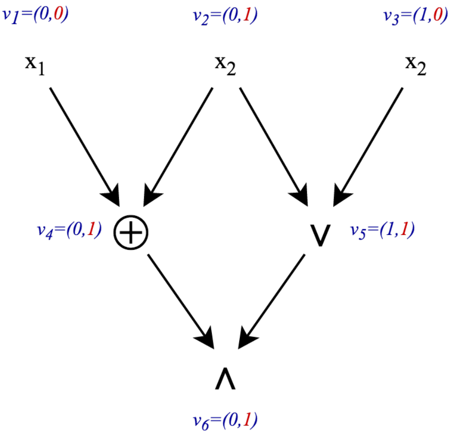
**(Generalized) Differential Computation Analysis (DCA):**




$x^{(1)} = (0,0,1)$

# Attacks II

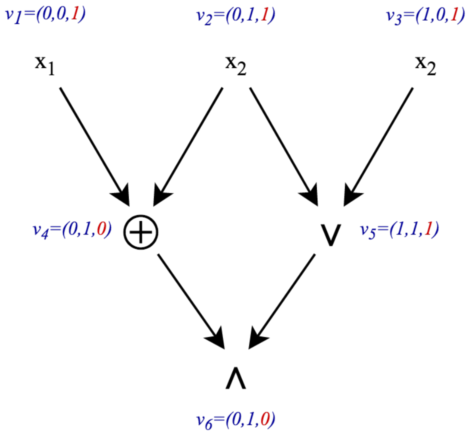
**(Generalized) Differential Computation Analysis (DCA):**




$x^{(1)} = (0,0,1)$

$x^{(2)} = (0,1,0)$

**(Generalized) Differential Computation Analysis (DCA):**




$x^{(1)} = (0,0,1)$

$x^{(2)} = (0,1,0)$

$x^{(3)} = (1,1,1)$

# The Linear Algebra Attack (1)

- consider the Boolean masking (the **linear** decoder)
- matching with a predictable value $s$:
  a basic linear algebra problem:

$$M \times z = s, \qquad M = [v_1 \mid \ldots \mid v_n]$$

# The Linear Algebra Attack (1)

- consider the Boolean masking (the **linear** decoder)
- matching with a predictable value $s$:
  a basic linear algebra problem:

$$M \times z = s, \qquad M = [v_1 \mid \ldots \mid v_n]$$

- $v_i$ is the vector of values computed in the node $i$ of the circuit
- $z$ is a vector indicating locations of shares among nodes of the circuit
- higher-order masking does not help...

# The Linear Algebra Attack (2)

Generalizations:

- **nonlinear decoders**, through linearization technique
- **approximately linear decoders**, through LPN algorithms

# The Linear Algebra Attack (2)

Generalizations:

- **nonlinear decoders**, through linearization technique
- **approximately linear decoders**, through LPN algorithms
- **semi-linear decoders:**
  1. assume $s \cdot r$ is computed/shared in the circuit, where
  2. $s$ is a predictable value
  3. $r$ is unpredictable (pseudorandom, $\approx$ uniform)

# The Linear Algebra Attack (2)

Generalizations:

- **nonlinear decoders**, through linearization technique
- **approximately linear decoders**, through LPN algorithms
- **semi-linear decoders:**
  1. assume $s \cdot r$ is computed/shared in the circuit, where
  2. $s$ is a predictable value
  3. $r$ is unpredictable (pseudorandom, $\approx$ uniform)
  4. choose plaintexts $p_1, \ldots, p_D$ such that:
     $s(p_i) = 0 \quad$ for $1 \leq i \leq D-1$,
     $s(p_i) = 1 \quad$ for $i = D$.
  5. $s \cdot r$ will be equal to $(0, 0, \ldots, 0, 1)$ with $\Pr = 1/2$
  6. if $s$ is guessed wrong, such vector is unlikely to be a solution

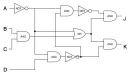
# Plan

# Our Framework: Two Components

## Value Hiding

00101010111010010010101011101001
10101010100101010001001010101011
...
10000010011000000010101010001001
011000011100000010010101011101110

## Structure Hiding

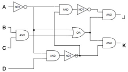


# Our Framework: Two Components

**Value Hiding**

00101010111010010010101011101001
10101010100101010001001010101011
...
100000100110000000010101010001001
01100001110000001001010101011101110

**Structure Hiding**




1. DCA side-channel attack
2. (new) linear algebra attack

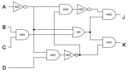
# Our Framework: Two Components

### Value Hiding

00101010111010010010101011101001
10101010100101010001001010101011
...
10000010011000000001010101010001001
01100001110000010010101011101110

1. DCA side-channel attack
2. (new) linear algebra attack

### Structure Hiding




1. circuit analysis / simplification
2. fault injections
3. pseudorandomness removal
4. etc.

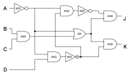
# Our Framework: Two Components

**Value Hiding**

00101010111010010010101011101001
10101010100101010001001010101011
...
10000010011000000001010101000100 1
01100001110000010010101011101110

1. DCA side-channel attack
2. (new) linear algebra attack

**Structure Hiding**




1. circuit analysis / simplification
2. fault injections
3. pseudorandomness removal
4. etc.

(hopefully) easier to solve independently

# Value Hiding

**Our solution for value hiding:**

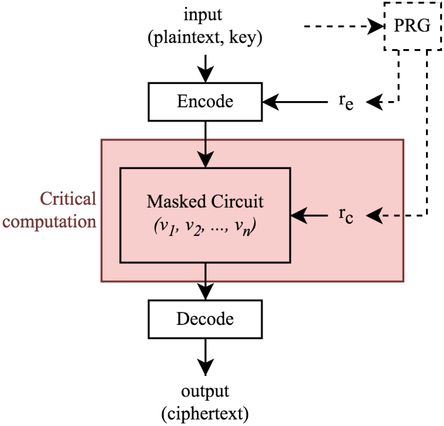
1. non-linear masking (vs linear algebra attack)
2. classic linear masking (vs DCA correlation attack)
3. provable security against the linear algebra attack

# Plan

# Algebraic Security (1/2)

Security Model:

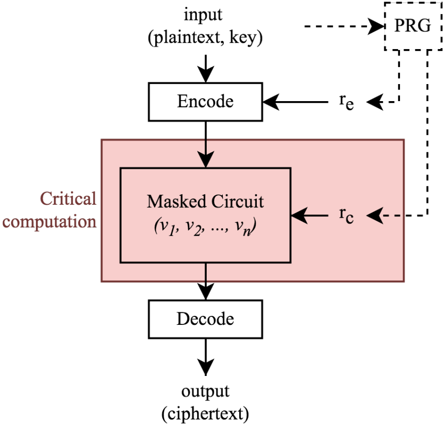
1. random bits allowed
   - as in classic masking
   - model unpredictability
   - in WB impl. as **pseudorandom**



# Algebraic Security (1/2)

Security Model:

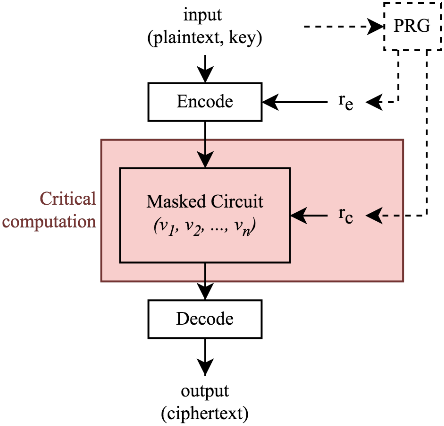
1. random bits allowed
   - as in classic masking
   - model unpredictability
   - in WB impl. as **pseudorandom**
2. **Goal:** any $f \in span\{v_i\}$ is unpredictable



# Algebraic Security (1/2)

Security Model:

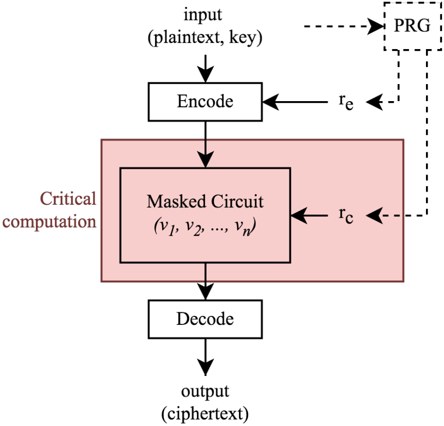
1. random bits allowed
   - as in classic masking
   - model unpredictability
   - in WB impl. as **pseudorandom**
2. **Goal:**
   any $f \in span\{v_i\}$ is unpredictable
3. **isolated** from obfuscation problems



# Algebraic Security (2/2)

Adversary:

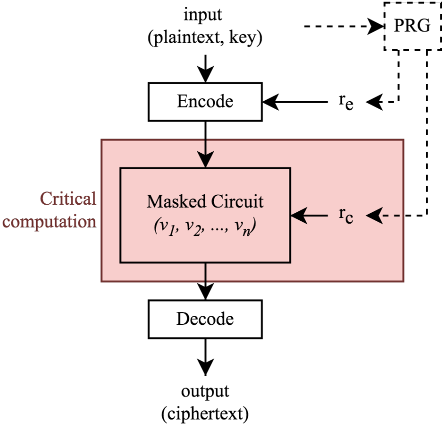
1. chooses plaintext/key pairs



# Algebraic Security (2/2)

Adversary:

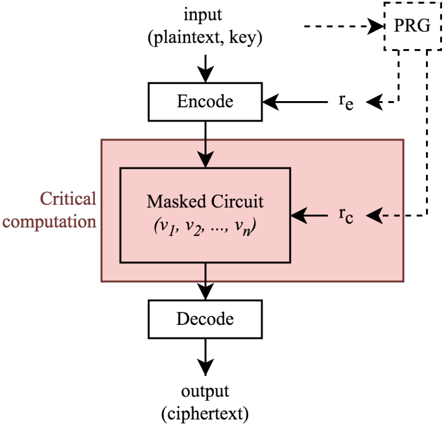
1. chooses plaintext/key pairs
2. chooses $f \in span\{v_i\}$



# Algebraic Security (2/2)

Adversary:

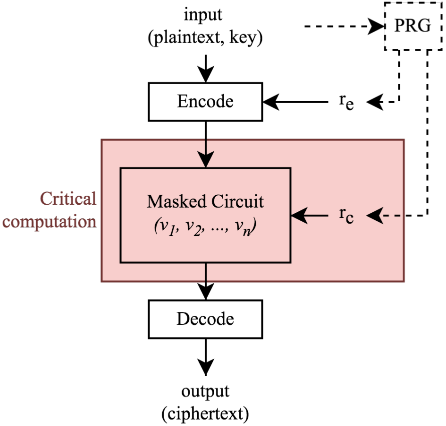
1. chooses plaintext/key pairs
2. chooses $f \in span\{v_i\}$
3. tries to **predict** values of this function (i.e. before random bits are sampled)



# Algebraic Security (2/2)

Adversary:

1. chooses plaintext/key pairs
2. chooses $f \in span\{v_i\}$
3. tries to **predict** values of this function (i.e. before random bits are sampled)
4. succeeds, if only $f$ matches



# Algebraic Security (3/3)

**Proposition**

Let $F = \{f(x,\cdot,\cdot) \mid f(x, r_e, r_c) \in span\{v_i\},\ x \in \mathbb{F}_2^N\}$.
Let $\varepsilon = \max_{f\in F} bias(f), e = -\log_2(1/2+\varepsilon)$.
Then for any adversary $\mathcal{A}$ choosing $Q$ inputs

$$\text{Adv}[\mathcal{A}] \leq min(2^{Q-|r_c|}, 2^{-eQ}).$$

# Algebraic Security (3/3)

## Proposition

Let $F = \{f(x,\cdot,\cdot) \mid f(x,r_e,r_c) \in span\{v_i\},\ x \in \mathbb{F}_2^N\}$.
Let $\varepsilon = \max_{f\in F} bias(f), e = -\log_2(1/2+\varepsilon)$.
Then for any adversary $\mathcal{A}$ choosing $Q$ inputs

$$\mathrm{Adv}[\mathcal{A}] \leq min(2^{Q-|r_c|}, 2^{-eQ}).$$

## Corollary

Let $k$ be a positive integer. Then for any adversary $\mathcal{A}$

$$\mathrm{Adv}[\mathcal{A}] \leq 2^{-k} \text{ if } e > 0 \text{ and } |r_c| \geq k \cdot (1 + \frac{1}{e}).$$

# Algebraic Security (3/3)

**Proposition**

Let $F = \{f(x, \cdot, \cdot) \mid f(x, r_e, r_c) \in span\{v_i\},\ x \in \mathbb{F}_2^N\}$.
Let $\varepsilon = \max_{f \in F} bias(f), e = -\log_2(1/2 + \varepsilon)$.
Then for any adversary $\mathcal{A}$ choosing $Q$ inputs

$$\mathrm{Adv}[\mathcal{A}] \leq min(2^{Q-|r_c|}, 2^{-eQ}).$$

**Corollary**

Let $k$ be a positive integer. Then for any adversary $\mathcal{A}$

$$\mathrm{Adv}[\mathcal{A}] \leq 2^{-k} \text{ if } e > 0 \text{ and } |r_c| \geq k \cdot (1 + \frac{1}{e}).$$

**Information-theoretic security**

# Minimalist Quadratic Masking Scheme (MQMS)

Masking scheme:

- set of **gadgets**
- provably secure **composition**

**function** Decode$(a, b, c)$
  **return** $ab \oplus c$

**function** EvalXOR$((a, b, c), (d, e, f), (r_a, r_b, r_c), (r_d, r_e, r_f))$
  $(a, b, c) \leftarrow$ Refresh$((a, b, c), (r_a, r_b, r_c))$
  $(d, e, f) \leftarrow$ Refresh$((d, e, f), (r_d, r_e, r_f))$
  $x \leftarrow a \oplus d$
  $y \leftarrow b \oplus e$
  $z \leftarrow c \oplus f \oplus ae \oplus bd$
  **return** $(x, y, z)$

**function** EvalAND$((a, b, c), (d, e, f), (r_a, r_b, r_c), (r_d, r_e, r_f))$
  $(a, b, c) \leftarrow$ Refresh$((a, b, c), (r_a, r_b, r_c))$
  $(d, e, f) \leftarrow$ Refresh$((d, e, f), (r_d, r_e, r_f))$
  $m_a \leftarrow bf \oplus r_c e$
  $m_d \leftarrow ce \oplus r_f b$
  $x \leftarrow ae \oplus r_f$
  $y \leftarrow bd \oplus r_c$
  $z \leftarrow am_a \oplus dm_d \oplus r_c r_f \oplus cf$
  **return** $(x, y, z)$

**function** Refresh$((a, b, c), (r_a, r_b, r_c))$
  $m_a \leftarrow r_a \cdot (b \oplus r_c)$
  $m_b \leftarrow r_b \cdot (a \oplus r_c)$
  $r_c \leftarrow m_a \oplus m_b \oplus (r_a \oplus r_c)(r_b \oplus r_c) \oplus r_c$
  $a \leftarrow a \oplus r_a$
  $b \leftarrow b \oplus r_b$
  $c \leftarrow c \oplus r_c$
  **return** $(a, b, c)$

# Minimalist Quadratic Masking Scheme (MQMS)

Masking scheme:

- set of **gadgets**
- provably secure **composition**
- quadratic decoder: $(a, b, c) \mapsto ab \oplus c$

**function** Decode($a, b, c$)
  **return** $ab \oplus c$

**function** EvalXOR($(a, b, c), (d, e, f), (r_a, r_b, r_c), (r_d, r_e, r_f)$)
  $(a, b, c) \leftarrow$ Refresh($(a, b, c), (r_a, r_b, r_c)$)
  $(d, e, f) \leftarrow$ Refresh($(d, e, f), (r_d, r_e, r_f)$)
  $x \leftarrow a \oplus d$
  $y \leftarrow b \oplus e$
  $z \leftarrow c \oplus f \oplus ae \oplus bd$
  **return** $(x, y, z)$

**function** EvalAND($(a, b, c), (d, e, f), (r_a, r_b, r_c), (r_d, r_e, r_f)$)
  $(a, b, c) \leftarrow$ Refresh($(a, b, c), (r_a, r_b, r_c)$)
  $(d, e, f) \leftarrow$ Refresh($(d, e, f), (r_d, r_e, r_f)$)
  $m_a \leftarrow bf \oplus r_c e$
  $m_d \leftarrow ce \oplus r_f b$
  $x \leftarrow ae \oplus r_f$
  $y \leftarrow bd \oplus r_c$
  $z \leftarrow am_a \oplus dm_d \oplus r_c r_f \oplus cf$
  **return** $(x, y, z)$

**function** Refresh($(a, b, c), (r_a, r_b, r_c)$)
  $m_a \leftarrow r_a \cdot (b \oplus r_c)$
  $m_b \leftarrow r_b \cdot (a \oplus r_c)$
  $r_c \leftarrow m_a \oplus m_b \oplus (r_a \oplus r_c)(r_b \oplus r_c) \oplus r_c$
  $a \leftarrow a \oplus r_a$
  $b \leftarrow b \oplus r_b$
  $c \leftarrow c \oplus r_c$
  **return** $(a, b, c)$

# Minimalist Quadratic Masking Scheme (MQMS)

Masking scheme:

- set of **gadgets**
- provably secure **composition**
- quadratic decoder: $(a, b, c) \mapsto ab \oplus c$
- first-order protection

**function** Decode($a, b, c$)
    **return** $ab \oplus c$

**function** EvalXOR($(a, b, c), (d, e, f), (r_a, r_b, r_c), (r_d, r_e, r_f)$)
    $(a, b, c) \leftarrow$ Refresh($(a, b, c), (r_a, r_b, r_c)$)
    $(d, e, f) \leftarrow$ Refresh($(d, e, f), (r_d, r_e, r_f)$)
    $x \leftarrow a \oplus d$
    $y \leftarrow b \oplus e$
    $z \leftarrow c \oplus f \oplus ae \oplus bd$
    **return** $(x, y, z)$

**function** EvalAND($(a, b, c), (d, e, f), (r_a, r_b, r_c), (r_d, r_e, r_f)$)
    $(a, b, c) \leftarrow$ Refresh($(a, b, c), (r_a, r_b, r_c)$)
    $(d, e, f) \leftarrow$ Refresh($(d, e, f), (r_d, r_e, r_f)$)
    $m_a \leftarrow bf \oplus r_c e$
    $m_d \leftarrow ce \oplus r_f b$
    $x \leftarrow ae \oplus r_f$
    $y \leftarrow bd \oplus r_c$
    $z \leftarrow am_a \oplus dm_d \oplus r_c r_f \oplus cf$
    **return** $(x, y, z)$

**function** Refresh($(a, b, c), (r_a, r_b, r_c)$)
    $m_a \leftarrow r_a \cdot (b \oplus r_c)$
    $m_b \leftarrow r_b \cdot (a \oplus r_c)$
    $r_c \leftarrow m_a \oplus m_b \oplus (r_a \oplus r_c)(r_b \oplus r_c) \oplus r_c$
    $a \leftarrow a \oplus r_a$
    $b \leftarrow b \oplus r_b$
    $c \leftarrow c \oplus r_c$
    **return** $(a, b, c)$

# MQMS Security

Security:

1. algorithm to verify that bias $\neq 1/2$
2. max. degree on $r$: 4

**function** Decode($a, b, c$)
  **return** $ab \oplus c$

**function** EvalXOR($(a, b, c), (d, e, f), (r_a, r_b, r_c), (r_d, r_e, r_f)$)
  $(a, b, c) \leftarrow$ Refresh($(a, b, c), (r_a, r_b, r_c)$)
  $(d, e, f) \leftarrow$ Refresh($(d, e, f), (r_d, r_e, r_f)$)
  $x \leftarrow a \oplus d$
  $y \leftarrow b \oplus e$
  $z \leftarrow c \oplus f \oplus ae \oplus bd$
  **return** $(x, y, z)$

**function** EvalAND($(a, b, c), (d, e, f), (r_a, r_b, r_c), (r_d, r_e, r_f)$)
  $(a, b, c) \leftarrow$ Refresh($(a, b, c), (r_a, r_b, r_c)$)
  $(d, e, f) \leftarrow$ Refresh($(d, e, f), (r_d, r_e, r_f)$)
  $m_a \leftarrow bf \oplus r_c e$
  $m_d \leftarrow ce \oplus r_f b$
  $x \leftarrow ae \oplus r_f$
  $y \leftarrow bd \oplus r_c$
  $z \leftarrow am_a \oplus dm_d \oplus r_c r_f \oplus cf$
  **return** $(x, y, z)$

**function** Refresh($(a, b, c), (r_a, r_b, r_c)$)
  $m_a \leftarrow r_a \cdot (b \oplus r_c)$
  $m_b \leftarrow r_b \cdot (a \oplus r_c)$
  $r_c \leftarrow m_a \oplus m_b \oplus (r_a \oplus r_c)(r_b \oplus r_c) \oplus r_c$
  $a \leftarrow a \oplus r_a$
  $b \leftarrow b \oplus r_b$
  $c \leftarrow c \oplus r_c$
  **return** $(a, b, c)$

# MQMS Security

Security:

1. algorithm to verify that bias $\neq 1/2$
2. max. degree on $r$: 4

$\Rightarrow$ bias $\leq 7/16$

for 80-bit security we need $|r_c| \geq 940$

**function** Decode($a, b, c$)
  **return** $ab \oplus c$

**function** EvalXOR($(a,b,c),(d,e,f),(r_a,r_b,r_c),(r_d,r_e,r_f)$)
  $(a,b,c) \leftarrow$ Refresh($(a,b,c),(r_a,r_b,r_c)$)
  $(d,e,f) \leftarrow$ Refresh($(d,e,f),(r_d,r_e,r_f)$)
  $x \leftarrow a \oplus d$
  $y \leftarrow b \oplus e$
  $z \leftarrow c \oplus f \oplus ae \oplus bd$
  **return** $(x,y,z)$

**function** EvalAND($(a,b,c),(d,e,f),(r_a,r_b,r_c),(r_d,r_e,r_f)$)
  $(a,b,c) \leftarrow$ Refresh($(a,b,c),(r_a,r_b,r_c)$)
  $(d,e,f) \leftarrow$ Refresh($(d,e,f),(r_d,r_e,r_f)$)
  $m_a \leftarrow bf \oplus r_c e$
  $m_d \leftarrow ce \oplus r_f b$
  $x \leftarrow ae \oplus r_f$
  $y \leftarrow bd \oplus r_c$
  $z \leftarrow am_a \oplus dm_d \oplus r_c r_f \oplus cf$
  **return** $(x,y,z)$

**function** Refresh($(a,b,c),(r_a,r_b,r_c)$)
  $m_a \leftarrow r_a \cdot (b \oplus r_c)$
  $m_b \leftarrow r_b \cdot (a \oplus r_c)$
  $r_c \leftarrow m_a \oplus m_b \oplus (r_a \oplus r_c)(r_b \oplus r_c) \oplus r_c$
  $a \leftarrow a \oplus r_a$
  $b \leftarrow b \oplus r_b$
  $c \leftarrow c \oplus r_c$
  **return** $(a,b,c)$

# Implementation

Proof-of-concept masked AES-128

1. MQMS + 1-st order Boolean masking
2. 31,783 $\rightarrow$ 2,588,743 gates expansion (x81)
3. 16 Mb code / 1 Kb RAM / 0.05s per block on a laptop
4. (unoptimized)

github.com/cryptolu/whitebox

# Conclusions

**Conclusions:**

1. new attack methods ⇒ new constraints on a white-box impl.
2. new results on provable security for white-box model
3. new links with side-channel research

# Conclusions

## Conclusions:

1. new attack methods $\Rightarrow$ new constraints on a white-box impl.
2. new results on provable security for white-box model
3. new links with side-channel research

## Open problems and future work:

1. structure-hiding component
2. higher-order protection
3. analysis of LPN-based attacks
4. deeper study of the fault attacks
5. optimizations

# The End

ePrint 2018/049

github.com/cryptolu/whitebox

**Thank you!**