Magic pot: Algebraic cryptanalysis of full AIM2

Alex Biryukov Pablo García Fernández Aleksei Udovenko

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DCS and SnT, University of Luxembourg

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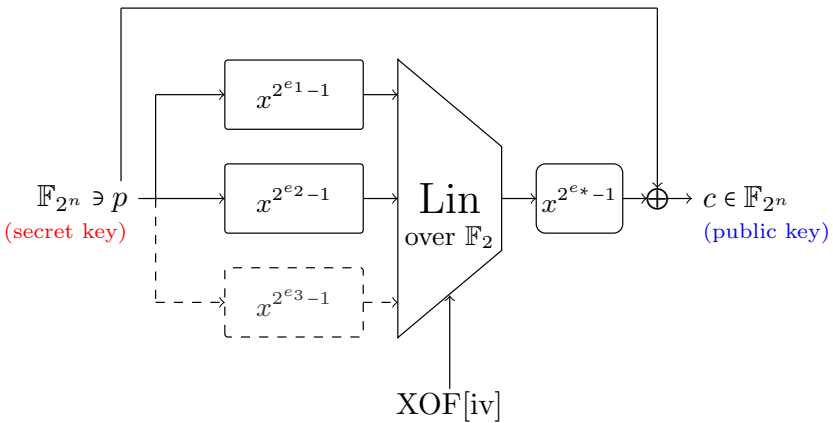
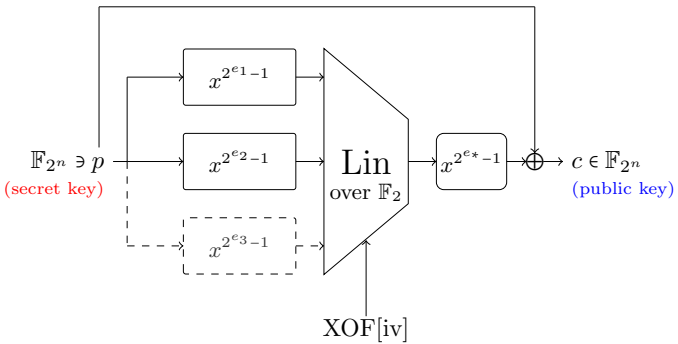
AIM1

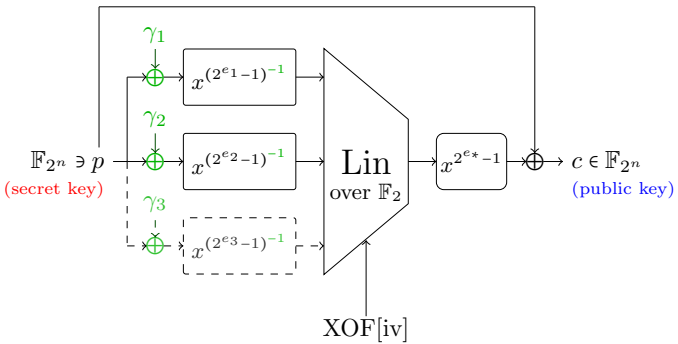

$\mathbb{F}_{2^n} \ni p$
(secret key)
$x^{2^{e_1}-1}$
$x^{2^{e_2}-1}$
$x^{2^{e_3}-1}$
Lin
over $\mathbb{F}_2$
$x^{2^{e_*}-1}$
$c \in \mathbb{F}_{2^n}$
(public key)
XOF[iv]

AIM1



Version	n	ℓ	e_1	e_2	e_3	e_*
AIM-I	128	2	3	27	-	5
AIM-III	192	2	5	29	-	7
AIM-V	256	3	3	53	7	5

AIM2



Version	n	ℓ	e_1	e_2	e_3	e_*
AIM2-I	128	2	49	91	-	3
AIM2-III	192	2	17	47	-	5
AIM2-V	256	3	11	141	7	3

Plan

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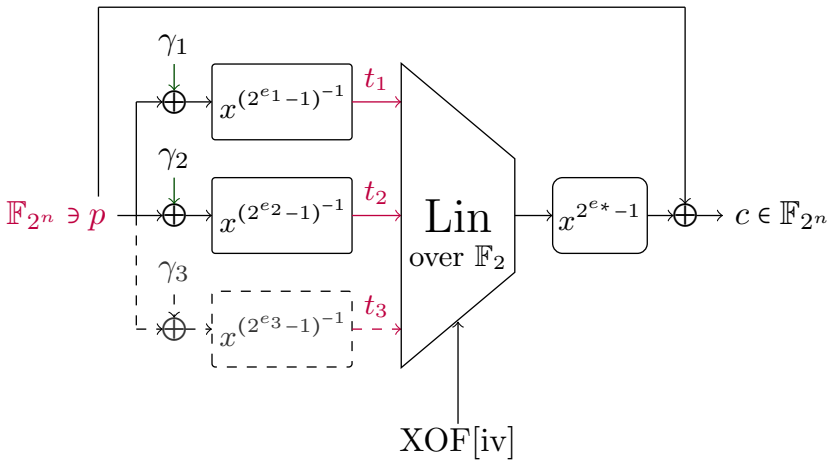
Mixed-order modeling

Path of elimination

Path of determination

Results and conclusions

System of equations (1/2)



System of equations (2/2)

$$(p + \gamma_1)^{(2^{e_1}-1)^{-1}} = t_1$$

System of equations (2/2)

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System of equations (2/2)

$$\begin{cases} (p+\gamma_1)\cdot t_1 = t_1^{2^{e_1}} \\ (p+\gamma_2)\cdot t_2 = t_2^{2^{e_2}} \\ (p+\gamma_3)\cdot t_3 = t_3^{2^{e_3}} \quad \text{(when } \ell = 3\text{)} \\ (p+c)\cdot \mathrm{Lin}(\mathrm{t}_1,\ldots,\mathrm{t}_\ell) = \mathrm{Lin}(\mathrm{t}_1,\ldots,\mathrm{t}_\ell)^{2^{\mathrm{e}_*}} \end{cases}$$

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Monomials: $1,\ t_j^{2^i},\ pt_j^{2^i}$ $(2\ell n + 1$ in total)

Equations: $1 + \ell$ (3 or 4)

Multiplying equations - Start cooking!

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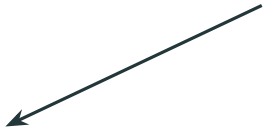

$$(p+\gamma_1)t_1 \times p^j = t_1^{2^{e_1}} \times p^j$$

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$$(p+\gamma_1)^{2^m} t_1^{2^m} = (t_1^{2^{e_1}})^{2^m}$$

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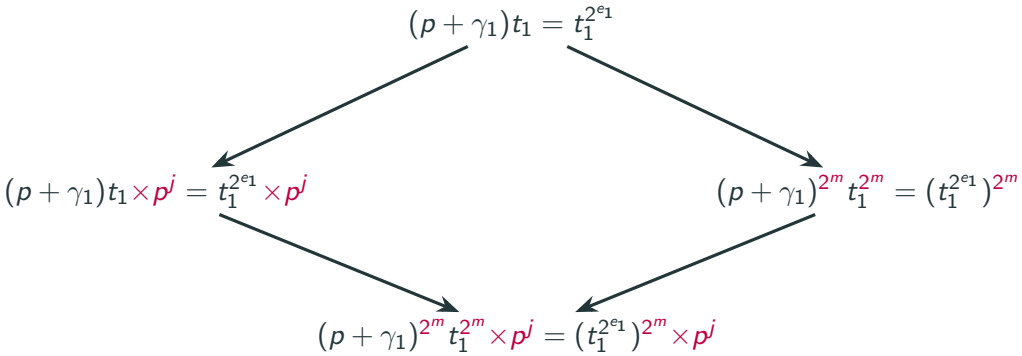

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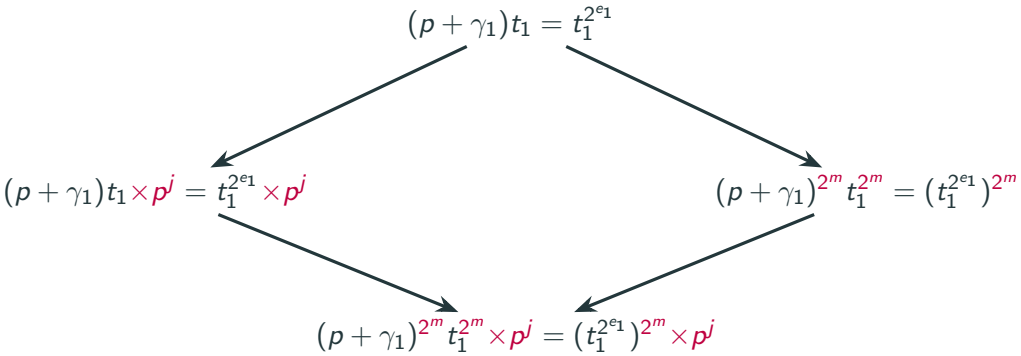

- Monomials $\{1, p, p^2, p^3, \ldots, p^d\} \times \{1, t_1, t_1^2, t_1^4, \ldots, t_1^{2^{n-1}}, t_2, \ldots, t_\ell^{2^{n-1}}\}$

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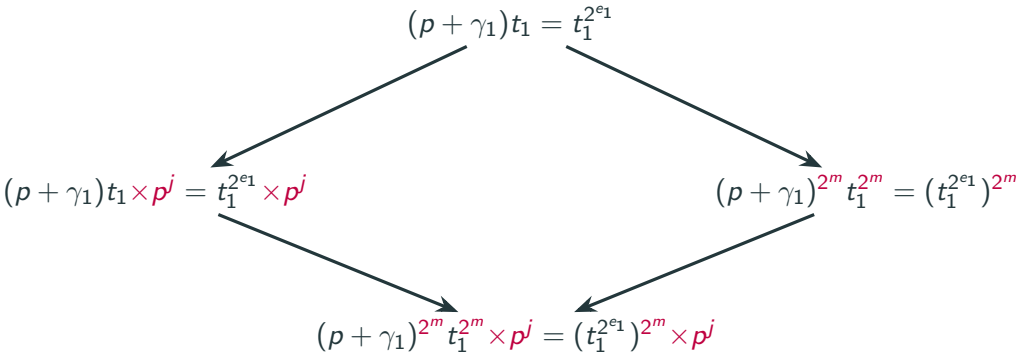

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- Set $d = 2^M$, $0 \leq m \leq M$, $0 \leq j \leq 2^M - 2^m$

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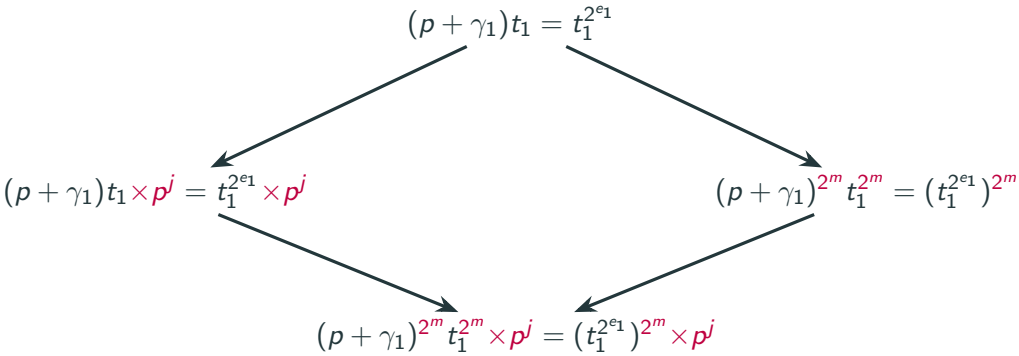

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- Set $d = 2^M$, $0 \leq m \leq M$, $0 \leq j \leq 2^M - 2^m$
- $M = \lceil \frac{\ell}{\ell+1} n \rceil$ ($\frac{2}{3}n$ or $\frac{3}{4}n$) $\Rightarrow$ #equations $\approx$ #monomials

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Sparse linearization (XL)

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Complexity using Wiedemann's method ($\omega = 2$): $\tilde{O}(2^{1.5n})$ for $\ell = 2$ ...

Polynomials

Idea: multiplications by p^j and linear combinations of equations

$\simeq$ multiplications by polynomials $f(p)$

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$\simeq$ multiplications by polynomials $f(p)$

$\Rightarrow$ let's work in $\mathbb{F}_{2^n}[p][t_1, \ldots, t_\ell]$!
(i.e., treat polynomial in p as scalars)[1]

[1] See also [Bariant, Boeuf, Lemoine, Ayala, Øygarden, Perrin, and Raddum 2024 CRYPTO], [Berthomieu, Neiger, and Din 2022 ISSAC]

System of equations

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\begin{cases}
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$\ell n + 1$ monomials $(1,\ t_1,\ t_1^2,\ \ldots,\ t_\ell^{2^{n-1}})$

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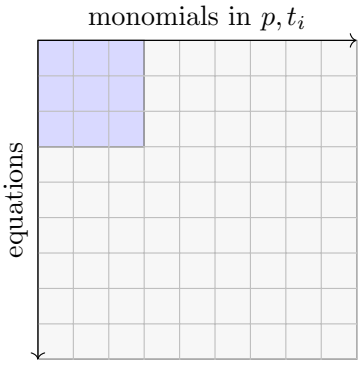
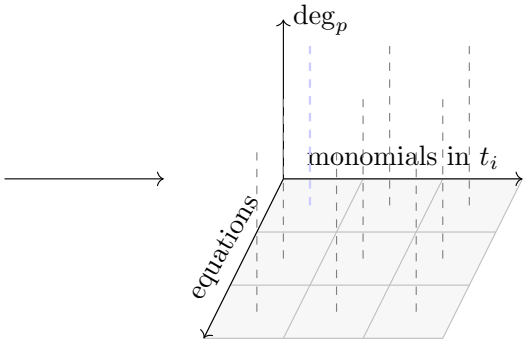


Linearized matrix over $\mathbb{F}_{2^n}$



Linearized matrix over $\mathbb{F}_{2^n}[p]$

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- Sufficient to get full rank at any fixed p!
- For example, $p = 0$ - the constant coefficients matrix ($\ell = 2$):
  - matrix size $(2n + 1) \times (2n + 1)$, over $\mathbb{F}_{2^n}$
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- Provably, also yes! Quite involved, but possible due to structured system.

Solving?

	1	t_1	t_1^2	...	$t_\ell^{2^{n-1}}$
f_1					
f_2					
f_3					
$\vdots$					
f_H					

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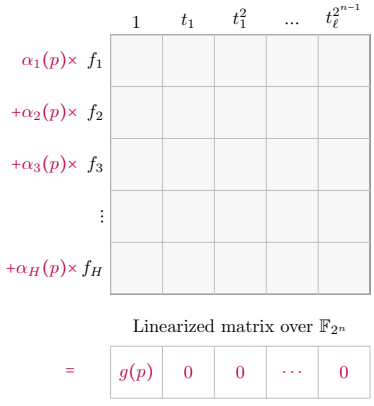
1
t_1
t_1^2
...
$t_\ell^{2^{n-1}}$
$\alpha_1(p)\times$ f_1
$+\alpha_2(p)\times$ f_2
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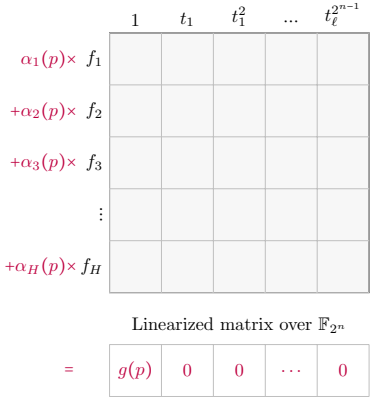
$+\alpha_H(p)\times$ f_H
Linearized matrix over $\mathbb{F}_{2^n}$
=
$g(p)$
0
0
$\cdots$
0

Solving?



$\Rightarrow g(p) = 0$, univariate root finding

Solving? Details

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- Overall complexity $\approx$ univariate root finding of degree $2^{2n/3}$ ($\ell = 2$)

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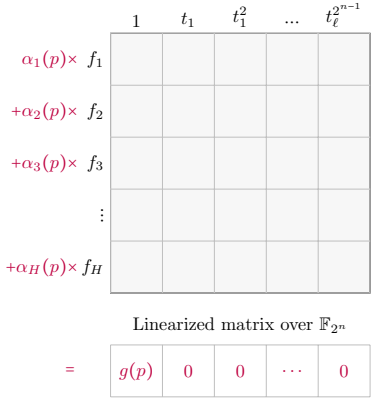

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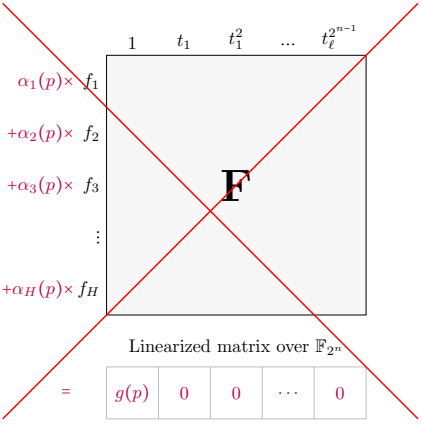

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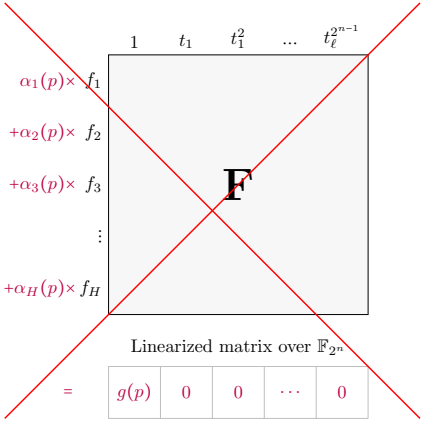

Observation: $g(p)$ divides $\det \mathbf{F}$!!!

Determination process

Consider equation system in $\mathbb{F}[x, y_1, \ldots, y_\ell]$.

Elegant and simple procedure:

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No Gröbner bases, multiplication matrix computations, etc.

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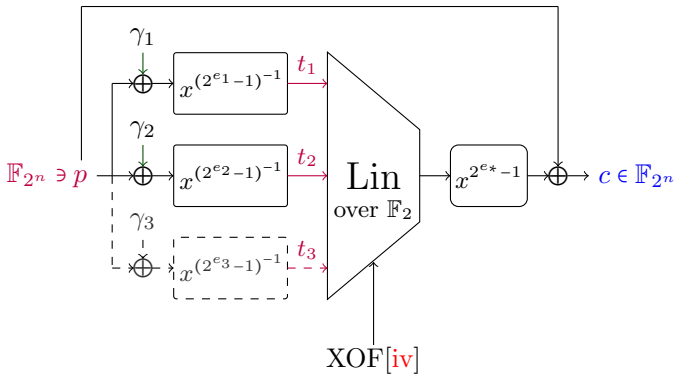
Results and conclusions

Reused-key and related-key models



Extended models for analysis:

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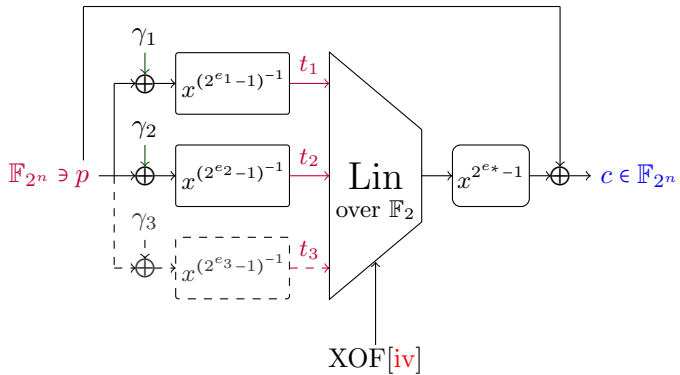

Extended models for analysis:

- Reuse key p with different IVs:
  $(p, \mathrm{iv}_1) \to c_1$
  $(p, \mathrm{iv}_2) \to c_1$
  $\vdots$
  $(p, \mathrm{iv}_L) \to c_L$

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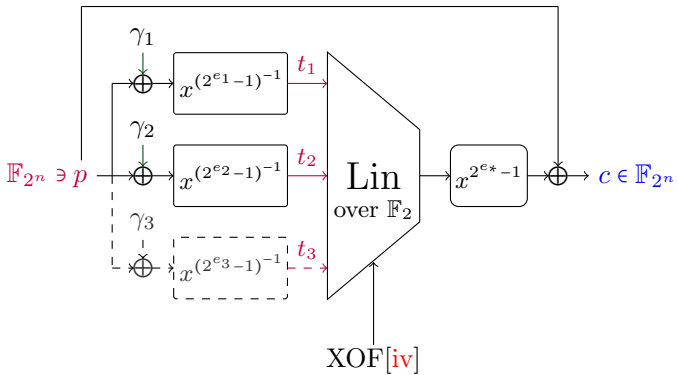
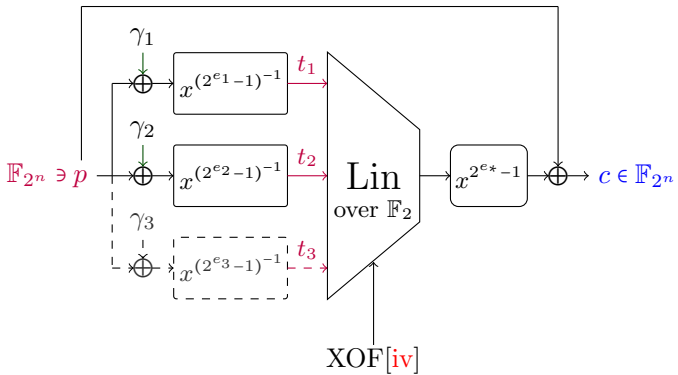


Extended models for analysis:

- Reuse key p with different IVs:
- Related-key variant ($\ell = 2$)
  - $(p, \mathrm{iv}_1) \to c_1$
  - $(L(p), \mathrm{iv}_1) \to c_2$
  - $(L^2(p), \mathrm{iv}_1) \to c_3$
  - $(L^3(p), \mathrm{iv}_2) \to c_4$

Reused-key and related-key models



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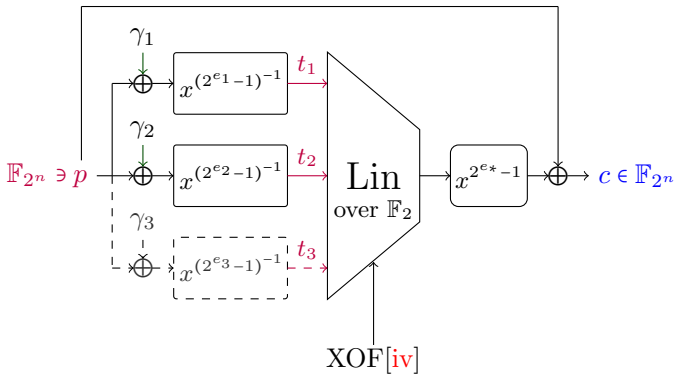

Extended models for analysis:

- Reuse key p with different IVs:
- Related-key variant ($\ell = 2$)
- Realistic? Potentially.
- Benchmarking and experimental verification!

Cryptanalysis results

Target	n	ℓ	Model	Time (Enc.)	Memory (b)
AIM2-I	128	2	Reuse, 109 IVs	$2^{20.60}$ (3s)	$2^{23.79}$
AIM2-I	128	2	Reuse, 15 IVs	$2^{31.44}$ (1h)	$2^{32.26}$
AIM2-I	128	2	Standard	$2^{103.68}$	$2^{100.33}$

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AIM2-III	192	2	Reuse, 20 IVs	$2^{35.31}$ (2 days)	$2^{36.22}$
AIM2-III	192	2	Standard	$2^{148.04}$	$2^{144.17}$
AIM2-V	256	3	Reuse, 94 IVs	$2^{29.38}$ (38 min)	$2^{30.34}$
AIM2-V	256	3	Standard	$2^{214.19}$	$2^{209.91}$

Conclusions

Theory

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Applications

- Cryptanalysis of full AIM2
- 3-round RAIN resists (not linear)
- **Q:** Revisit/simplify previous results?

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Solution?

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Solution? Ad-hoc mixed-order treatment!

Mixed-ordering

Two interpretations:

- *(Explicit)* Use change of basis

$$(t_i, t_i^2, t_i^4, \ldots, t_i^{2^{n-1}}) \in \mathbb{F}_{2^n}^n \quad \longleftrightarrow \quad (t_{i,0}, t_{i,1}, \ldots, t_{i,n-1}) \in \mathbb{F}_2^n$$

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- *(Implicit)* Manage the degrees by careful choice of multiplications/squarings:
  - degree in p at most d as integer
  - algebraic degree in $(t_1, t_2, \ldots, t_\ell)$ at most 1
  - $\Rightarrow$ monomial set $\{1, p, p^2, p^3, \ldots, p^d\} \otimes \{1, t_1, {t_1}^2, {t_1}^4, \ldots, {t_1}^{2^{n-1}}, t_2, \ldots, {t_\ell}^{2^{n-1}}\}$